Determination of the impedance of a multiperforated plate: an inverse problem

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Introduction

Development of new turbomachines by the aeronautic industry

- new ecological norms
- noise reduction
- reduction of fuel consumption

Classical turboreactors

- stoichiometric chemical reaction (nowadays well understood)

A possibility for the future turboreactors

- low density of fuel for a high density of air
- unstable combustion sensible to the acoustic phenomena inside the motor

A lot of parts of these turboreactors should be better understood. Among all of them there are the multiperforated plates

The physical problem

Why multiperforated plates in turbo engines?

- Temperature of a combustion chamber: $\sim 2000K$
- Temperature of the casing: $\sim 800K$
- Injection of "fresh" air from the casing to the combustion chamber to protect the boundary from the combustion (cooling film)

Importance of the acoustic perturbation

- For high ratio of fuel-air the combustion is unstable. The combustion can easily be perturbed by the acoustic
- The diameter ($\sim .5mm$) and spacing ($\sim 5mm$) of the holes are chosen to ensure the cooling and the solidity of the boundary.
- Holes with small diameters have an impact on the acoustic.

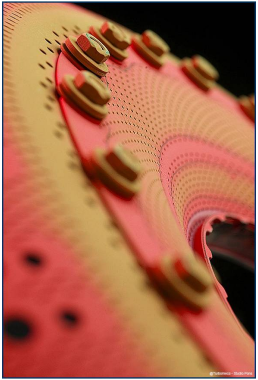
The limit of direct modeling

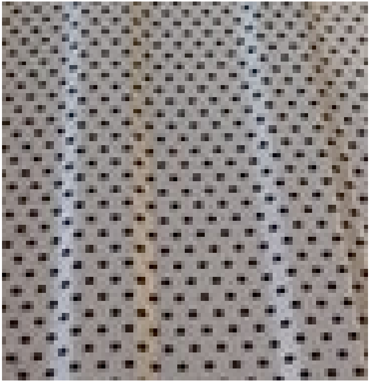
There are several papers dealing with the direct modeling

- purely acoustic models
  - Rayleigh 1896, Fok 1947, Tuck 1975, Bendali et al. 2013
- acoustic models with viscosity
  - Ingard 1957, Sanchez-Hubert 1982
- aeroacoustic models
  - Malmary 2000
- Direct numerical simulations
  - Eldredge et al.2007 Gullaud et al. 2009

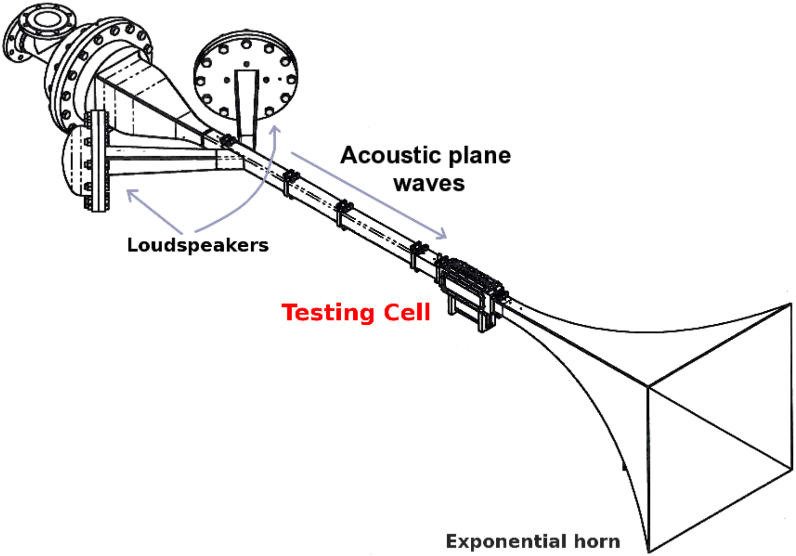
These papers do not give a good estimation of the energy dissipated close to the boundary layer.

Objective: Design a numerical tool which enables to determine the effective impedance of a multiperforated plate

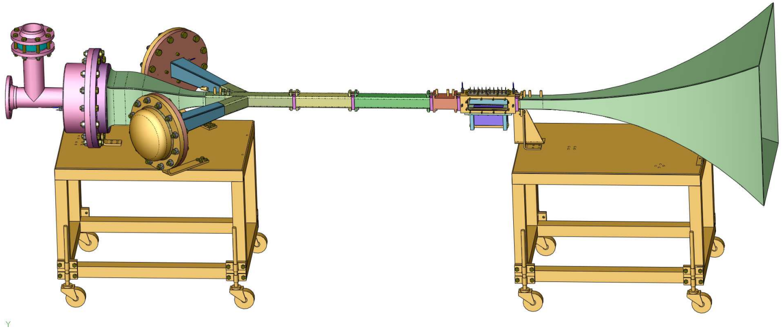




The acoustic bench B2A

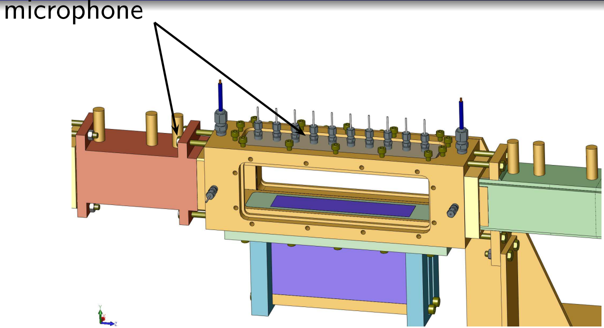


The acoustic bench B2A (aero-thermo acoustic bench)



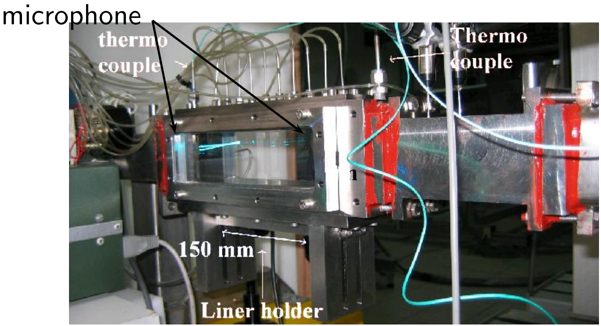
- Exponential horn to avoid parasitical reflexion
- Two loudspeakers to generate a plane entry wave
- An aero-module to generate eventually a mean flow
- A thermic module for experiments at high temperature

The testing cell



- Velocimetry laser through the glass
- pressure measurement via microphones
- measurement in frequency domain thanks to a postreatment
- level of error (measurement, ambiant noise, ...): less than 5%

The testing cell



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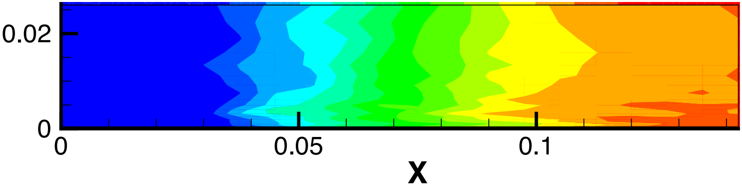
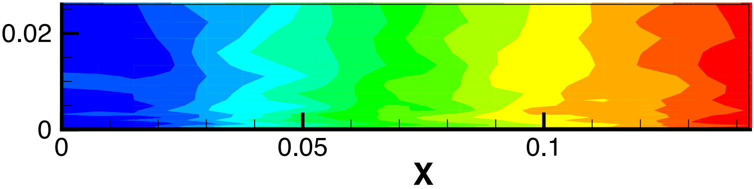
An example of measurements

- 400 Measurements of the speed velocity on the grid
- Real part of the x-velocity

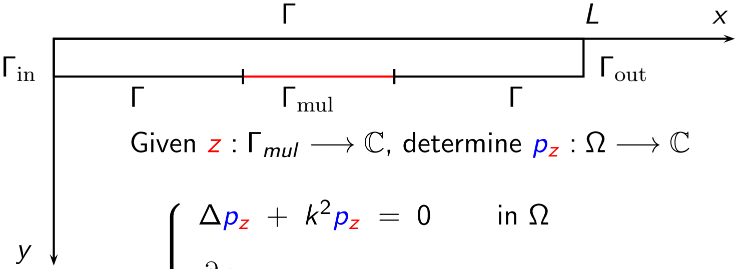


- Imaginary part of the x-velocity



Measurements realized at ONERA by E. Piot et al

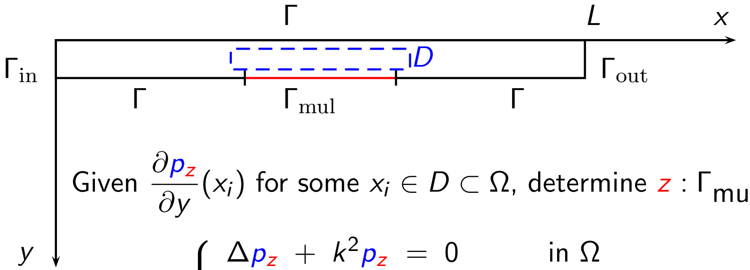
The direct problem



Given $z : \Gamma_{mul} \longrightarrow \mathbb{C}$, determine $p_z : \Omega \longrightarrow \mathbb{C}$

$$
\begin{cases}
\Delta p_z + k^2 p_z = 0 & \text{in } \Omega \\
\dfrac{\partial p_z}{\partial n} = 0 & \text{on } \Gamma \\
\dfrac{\partial p_z}{\partial n} - ik p_z = 1 & \text{on } \Gamma_{\text{in}} \\
\dfrac{\partial p_z}{\partial n} - ik p_z = 0 & \text{on } \Gamma_{\text{out}} \\
\dfrac{\partial p_z}{\partial n} - ik z p_z = 0 & \text{on } \Gamma_{\text{mul}}
\end{cases}
$$

This problem is nonlinear.



Given $\frac{\partial p_z}{\partial y}(x_i)$ for some $x_i \in D \subset \Omega$, determine $z : \Gamma_{\mathrm{mul}} \longrightarrow \mathbb{C}$

$$\begin{cases} \Delta p_z + k^2 p_z = 0 & \text{in } \Omega \\ \frac{\partial p_z}{\partial n} = 0 & \text{on } \Gamma \\ \frac{\partial p_z}{\partial n} - ikp_z = -2ik & \text{on } \Gamma_{\mathrm{in}} \\ \frac{\partial p_z}{\partial n} - ikp_z = 0 & \text{on } \Gamma_{\mathrm{out}} \\ \frac{\partial p_z}{\partial n} - ikzp_z = 0 & \text{on } \Gamma_{\mathrm{mul}} \end{cases}$$

Discretization of the impedance

Let us introduce a space of finite dimension

$$z(x) = \sum_{k=1}^{K} z^k \, w_k(x)$$

with w_k a basis of a classical finite dimensional space

$$\mathbb{P}_0(\Gamma_{\rm mul}) - \text{discont.} = \left\{ z \in C^0(\Gamma_{\rm mul}) \ : \ z_{|_{[x_k,x_{k+1}]}} = z^k \right\}$$

$$\mathbb{P}_1(\Gamma_{\rm mul}) - \text{cont.} = \left\{ z : \Gamma_{\rm mul} \longrightarrow \mathbb{C} \;\middle|\; z_{|_{[x_k,x_{k+1}]}} \in P_1 \text{ and } z(x_k) = z^k \right\}$$

The impedance function can be identified with a complex vector $z = (z^1, \cdots, z^K)$

A classical overdetermined problem

The inverse problem can be written as follows

$$\left\{ \begin{array}{lcl} \text{Find } z \in \mathbb{C}^K \text{ such that:} & & \\ \dfrac{\partial p_z}{\partial y}(x_1) & = & f_1 \\ \dfrac{\partial p_z}{\partial y}(x_2) & = & f_2 \\ \cdots & \cdots & \cdots \\ \dfrac{\partial p_z}{\partial y}(x_K) & = & f_K \end{array} \right. \tag{1}$$

with

- $f = (f_1, f_2, \cdots, f_I) \in \mathbb{C}^I$ a vector of complex measurements
- $z = (z_1, z_2, \cdots, z_K) \in \mathbb{C}^K$ a vector of complex parameters
- $(g_1, g_2, \cdot, g_I)$ is a vector of functions related to a model

A gradient based method

We introduce the classical functional

$$J(z) = \sum_{i=1}^{I} \left| \frac{\partial p_z}{\partial y}(x_i) - f_i \right|^2 \qquad (2)$$

A classical remark:

- The functional J is not differentiable with respect to z but with respect to its real a and its imaginary part b.

$$z = a + ib \quad \text{with} \quad z \in \mathbb{C}^J, \quad a \in \mathbb{R}^J \quad \text{and} \quad b \in \mathbb{R}^J$$

A descent algorithm

Let $z = a + ib$ and $K(a,b) = J(a+ib)$.
with the descent direction given by

$$\begin{cases} a_{n+1} = a_n - \dfrac{\alpha_n \, \nabla_a K(a_n, b_n)}{2} \\ b_{n+1} = b_n - \dfrac{\alpha_n \, \nabla_b K(a_n, b_n)}{2} \end{cases}$$

with

$$\begin{cases} \nabla_a K(a_n, b_n) = 2 \displaystyle\sum_{i=1}^{I} \Re\Big[\overline{\Big(\frac{\partial p_z}{\partial y}(x_i) - f_i \Big)} \nabla_z \Big(\frac{\partial p_z}{\partial y}(x_i) \Big) \Big] \\ \nabla_b K(a_n, b_n) = 2 \displaystyle\sum_{i=1}^{I} \Im\Big[\overline{\Big(\frac{\partial p_z}{\partial y}(x_i) - f_i \Big)} \nabla_z \Big(\frac{\partial p_z}{\partial y}(x_i) \Big) \Big] \end{cases}$$

A descent algorithm

In terms of z_n, it can be rewritten as

$$z_{n+1} = z_n - \alpha_n h_n \qquad (3)$$

with $h_n = \sum_{i=1}^{I} \overline{\left(\frac{\partial p_{z_n}}{\partial y}(x_i) - f_i\right)} \nabla_z \left(\frac{\partial p_{z_n}}{\partial y}(x_i)\right)$.

The parameter α_n is chosen in order to minimize J

$$\alpha_n = \underset{\alpha \in \mathbb{R}}{\text{argmin}}\, J(z_n + \alpha h_n) \quad \text{or} \quad \alpha_n = \underset{\alpha \in I_n \subset \mathbb{R}}{\text{argmin}}\, J(z_n + \alpha\, h_n) \quad (4)$$

To end the definition of the algorithm, it remains to compute

$$\nabla_z \left(\frac{\partial p_{z_n}}{\partial y}(x_i)\right) = \begin{pmatrix} \frac{\partial p^1_{z_n}}{\partial y}(x_i) \\ \cdots \\ \frac{\partial p^K_{z_n}}{\partial y}(x_i) \end{pmatrix} \quad \text{with} \quad p^m_{z_n} = \lim_{\varepsilon \to 0} \frac{p_{z_n + \varepsilon w_m} - p_{z_n}}{\varepsilon}$$

Toward the gradient

Deriving this expression with respect to z, we have

$$\begin{cases} \Delta p_z + k^2 p_z = 0 & \text{in } \Omega \\ \dfrac{\partial p_z}{\partial n} = 0 & \text{on } \Gamma \\ \dfrac{\partial p_z}{\partial n} - ik p_z = -2ik & \text{on } \Gamma_{\text{in}} \\ \dfrac{\partial p_z}{\partial n} - ik p_z = 0 & \text{on } \Gamma_{\text{out}} \\ \dfrac{\partial p_z}{\partial n} - ikz p_z = 0 & \text{on } \Gamma_{\text{mul}} \end{cases}$$

Toward the gradient

Deriving this expression with respect to z, we have

$$\begin{cases} \Delta p_z^m + k^2 p_z^m = 0 & \text{in } \Omega \\ \dfrac{\partial p_z}{\partial n} = 0 & \text{on } \Gamma \\ \dfrac{\partial p_z^m}{\partial n} - ik p_z^m = 0 & \text{on } \Gamma_{\text{in}} \\ \dfrac{\partial p_z^m}{\partial n} - ik p_z^m = 0 & \text{on } \Gamma_{\text{out}} \\ \dfrac{\partial p_z^m}{\partial n} - ikz p_z^m = ik\, w_m\, p_z & \text{on } \Gamma_{\text{mul}} \end{cases}$$

The last expression can be interpreted as a duality pairing

$$\frac{\partial p_z^m}{\partial y}(\mathbf{x}_i) = \left\langle -\frac{\partial \delta_{\mathbf{x}_i}}{\partial y}; p_z^m \right\rangle \quad \text{(Dirac generalized function)}$$

This leads to the introduction of an adjoint function

The adjoint function

$$\begin{cases} \Delta\phi_{z,m} + k^2\phi_{z,m} = -\dfrac{\partial\delta}{\partial y}(\mathbf{x}_i) & \text{in } \mathcal{D}'(\Omega) \quad \text{(Dirac generalized functic} \\ \dfrac{\partial\phi_{z,m}}{\partial n} = 0 & \text{on } \Gamma \\ \dfrac{\partial\phi_{z,m}}{\partial n} - ik\phi_{z,m} = 0 & \text{on } \Gamma_{\text{in}} \\ \dfrac{\partial\phi_{z,m}}{\partial n} - ik\phi_{z,m} = 0 & \text{on } \Gamma_{\text{out}} \\ \dfrac{\partial\phi_{z,m}}{\partial n} - ikz\phi_{z,m} = 0 & \text{on } \Gamma_{\text{mul}} \end{cases}$$

It becomes thanks to the Green Formula

$$\begin{cases} \frac{\partial p_z^m}{\partial y}(\mathbf{x}_i) & = \left\langle \Delta\phi_{z,m} + k^2\phi_{z,m}; p_z^m \right\rangle \\ & = -ik \displaystyle\int_{\Gamma_{\text{mul}}} p_z(x) w_m(x) \varphi \end{cases}$$

The adjoint function

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Numerical computation of the adjoint function

$$\begin{cases} \Delta\phi_{z,m} + k^2\phi_{z,m} = -\dfrac{\partial\delta}{\partial y}(\mathbf{x}_i) & \text{in } \mathcal{D}'(\Omega) \quad \text{(Dirac generalized functic} \\ \dfrac{\partial\phi_{z,m}}{\partial n} \;=\; 0 & \text{on } \Gamma \\ \dfrac{\partial\phi_{z,m}}{\partial n} - ik\phi_{z,m} \;=\; 0 & \text{on } \Gamma_{\text{in}} \\ \dfrac{\partial\phi_{z,m}}{\partial n} - ik\phi_{z,m} \;=\; 0 & \text{on } \Gamma_{\text{out}} \\ \dfrac{\partial\phi_{z,m}}{\partial n} - ikz\phi_{z,m} \;=\; 0 & \text{on } \Gamma_{\text{mul}} \end{cases}$$

This function is not regular but it can be decomposed into

$$\phi_{z,m} \;=\; \phi_{\star,m} + \phi_{z,m}^{\text{reg}}$$

with $\phi_{\star,m}(\mathbf{x}) \;=\; \dfrac{H_1^{(1)}(k\|\mathbf{x}-\mathbf{x}_i\|)}{4i}\frac{y-y_i}{\|\mathbf{x}-\mathbf{x}_i\|} \notin H^1(\Omega)$.

The regular part $\phi^{\mathrm{reg}}_{z,m}$

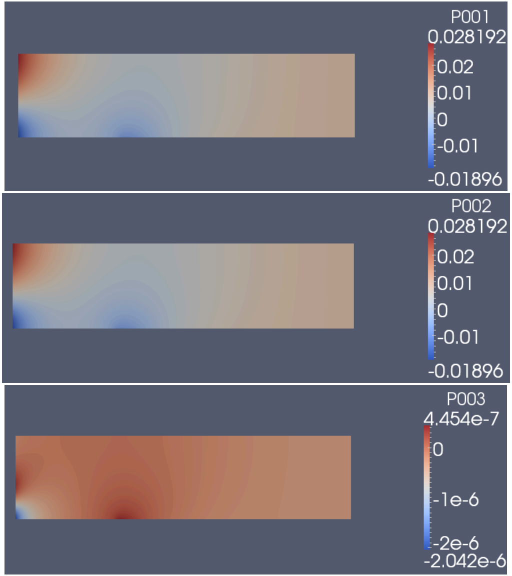
$$\begin{cases} \Delta\phi^{\mathrm{reg}}_{z,m} + k^2\phi^{\mathrm{reg}}_{z,m} = 0 & \text{in } \mathcal{D}'(\Omega) \\ \dfrac{\partial\phi^{\mathrm{reg}}_{z,m}}{\partial n} = -\dfrac{\partial\phi_{\star,m}}{\partial n} & \text{on } \Gamma \\ \dfrac{\partial\phi^{\mathrm{reg}}_{z,m}}{\partial n} - ik\phi^{\mathrm{reg}}_{z,m} = -\dfrac{\partial\phi_{\star,m}}{\partial n} + ik\phi_{\star,m} & \text{on } \Gamma_{\mathrm{in}} \\ \dfrac{\partial\phi^{\mathrm{reg}}_{z,m}}{\partial n} - ik\phi^{\mathrm{reg}}_{z,m} = -\dfrac{\partial\phi_{\star,m}}{\partial n} + ik\phi_{\star,m} & \text{on } \Gamma_{\mathrm{out}} \\ \dfrac{\partial\phi^{\mathrm{reg}}_{z,m}}{\partial n} - ikz\phi^{\mathrm{reg}}_{z,m} = -\dfrac{\partial\phi_{\star,m}}{\partial n} + ikz\phi_{\star,m} & \text{on } \Gamma_{\mathrm{mul}} \end{cases}$$

The function $\phi^{\mathrm{reg}}_{z,m} \in H^1(\Omega)$ is then numerically computed by any classical numerical method (Finite Element, Finite Difference, Discontinuous Galerkin, ...)

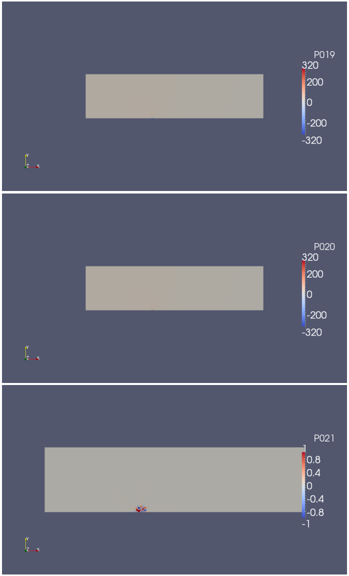
Remark: low extra cost, same matrix than for the direct problem

Two methods to compute the adjoint solutions

Two methods to compute the adjoint solutions



Two methods to compute the adjoint solutions



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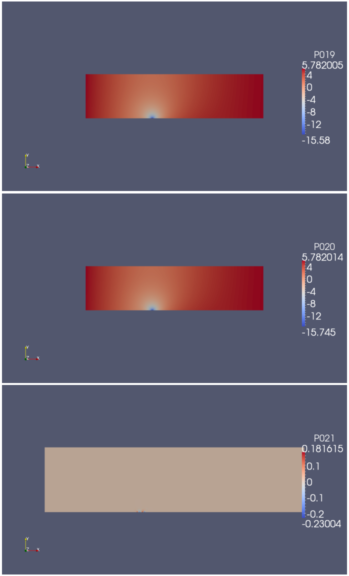

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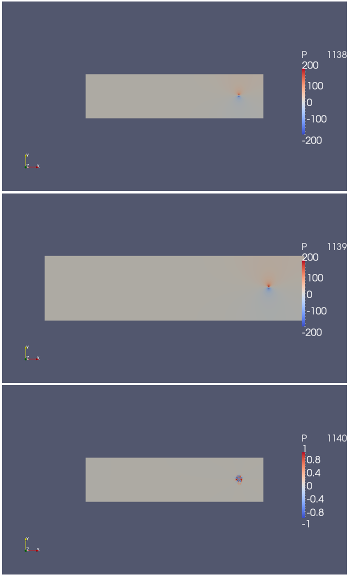

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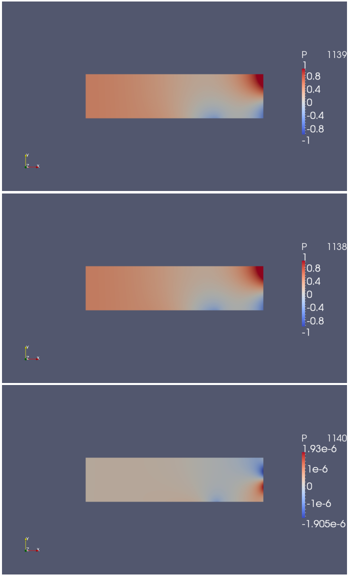


About the numerical simulation

The numerical code has been adapted from a library developed by the INRIA team Magique 3D (main original contributor: Julien Diaz).

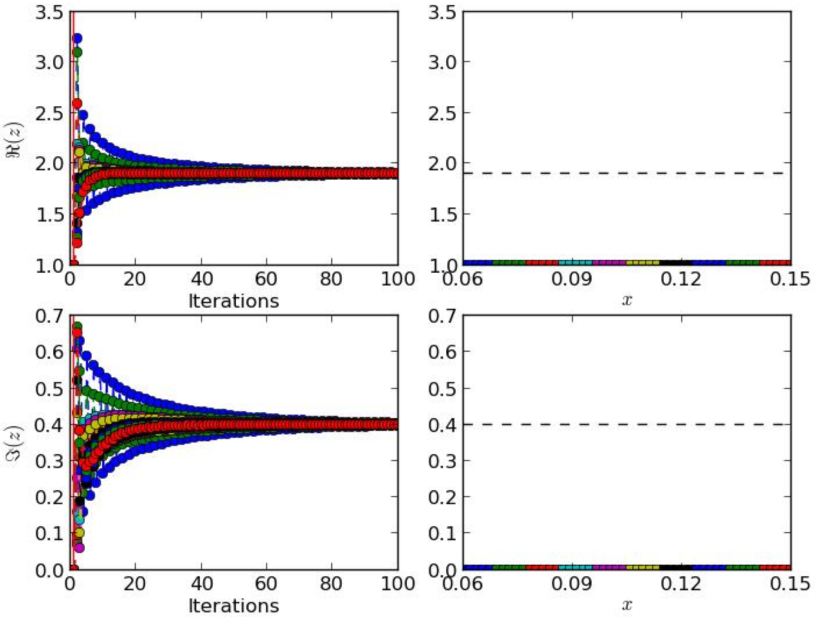
- Interior Penalty Discontinuous Galerkin Method
- a triangular mesh
- High order element: polynomial of degree 4
- Matrix inversion thanks to Mumps
- Most of the computation cost is concentrated in the computation of the LU factorization.
- Every iteration takes less than 10 seconds (on a simple laptop)

number of measurements: 19 in $x \times 20$ in $y = 380$

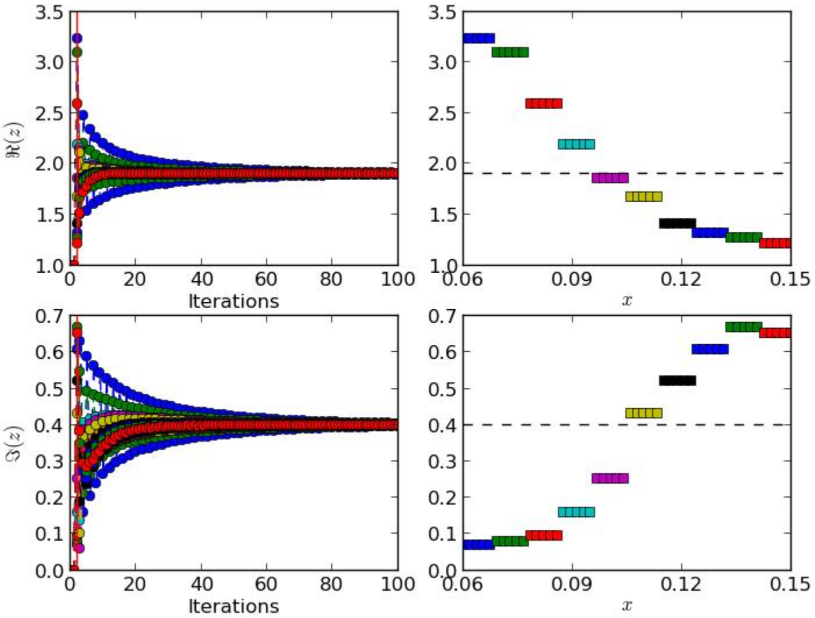
number of triangles: 1556

number of degrees of freedom: 23340

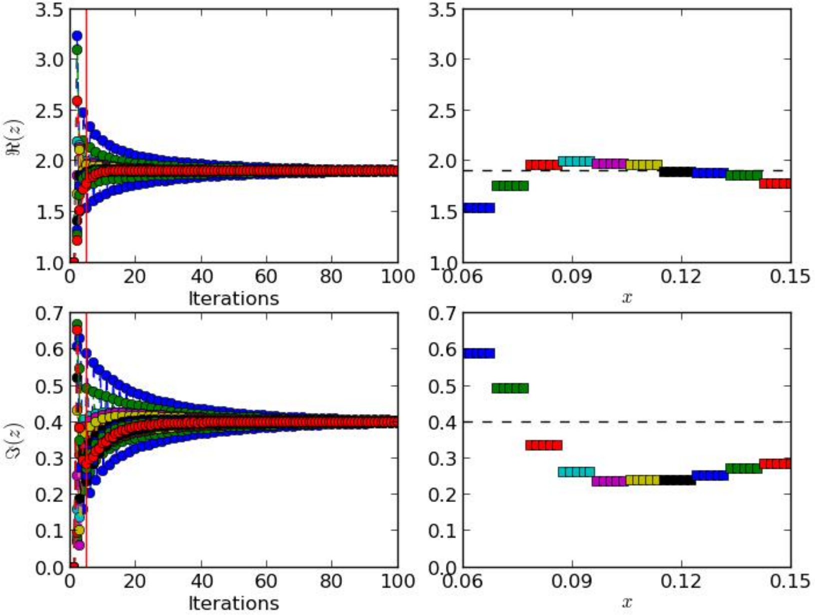
Numerical result. Iteration: 1 noise: 0%



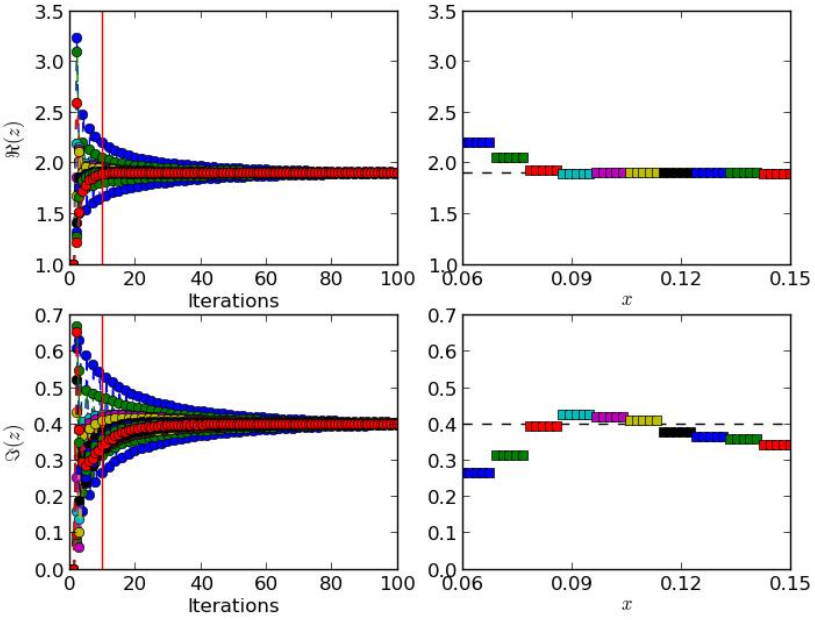
Numerical result. Iteration: 2 noise: 0%



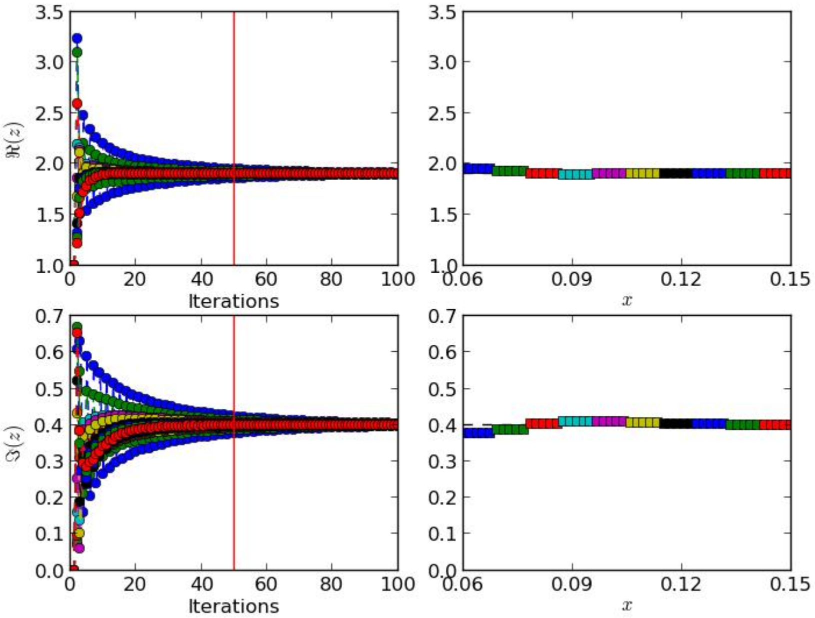
Numerical result. Iteration: 5 noise: 0%



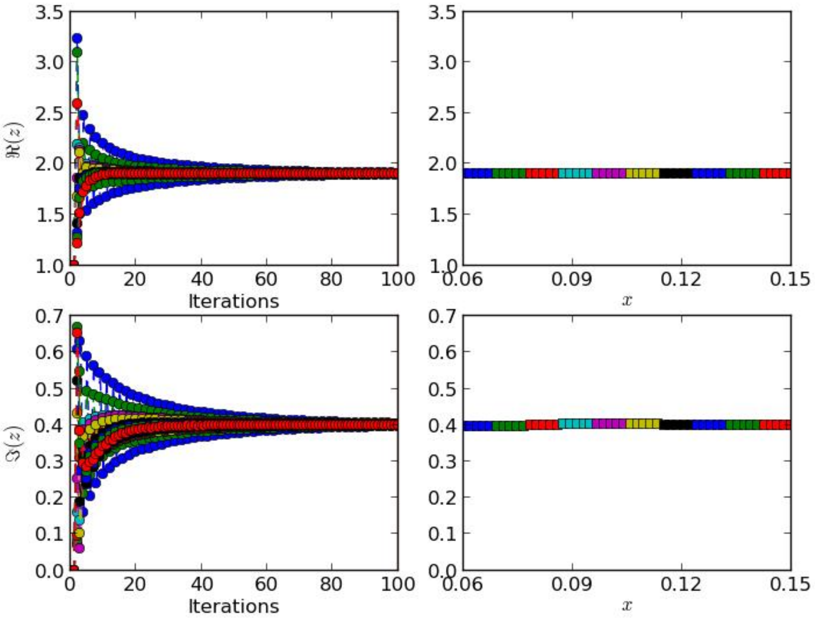
Numerical result. Iteration: 10 noise: 0%



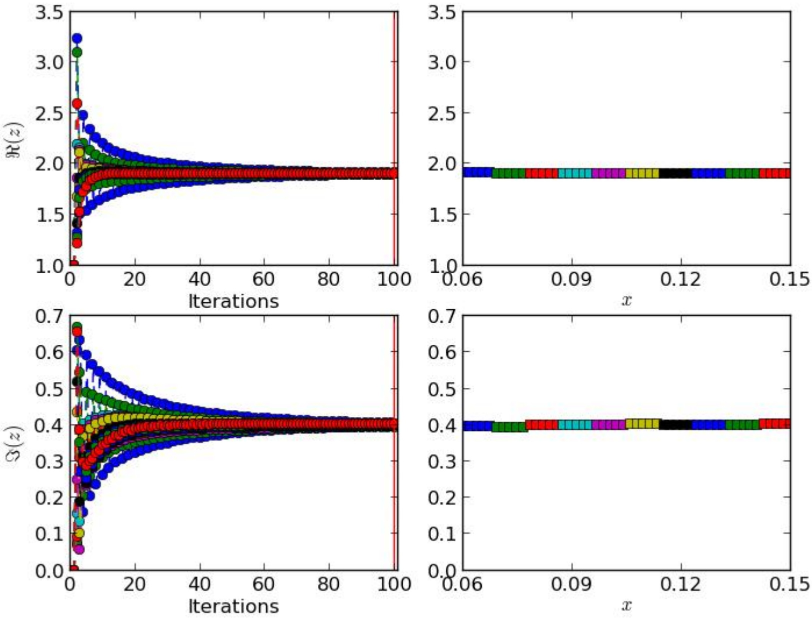
Numerical result. Iteration: 50 noise: 0%



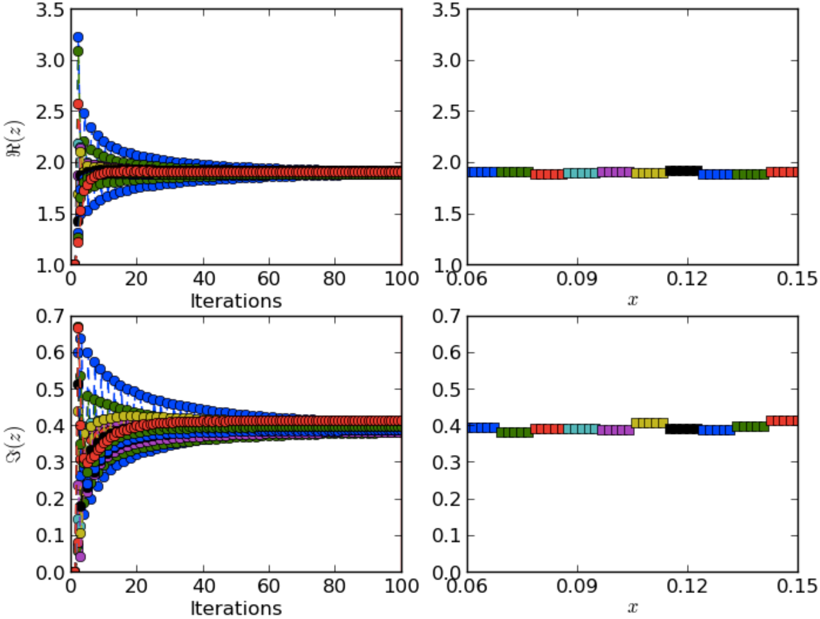
Numerical result. Iteration: 100 noise: 0%



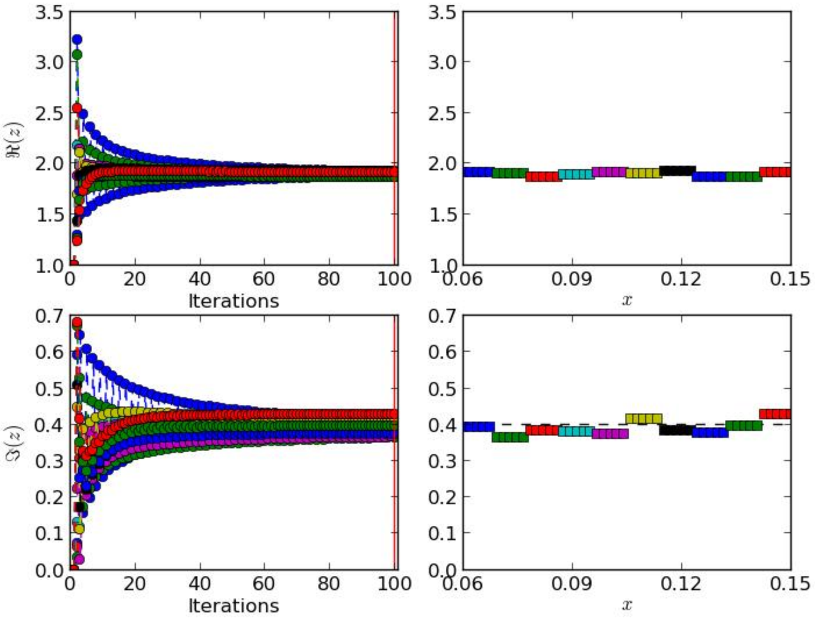
Numerical result. Iteration: 100 noise: 1%



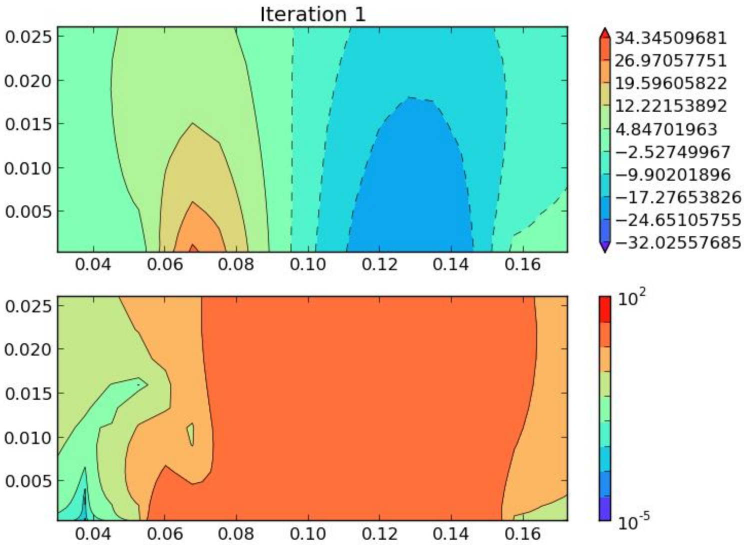
Numerical result. Iteration: 100 noise: 5%



Numerical result. Iteration: 100 noise: 10%



Convergence toward the measures, noise 0%



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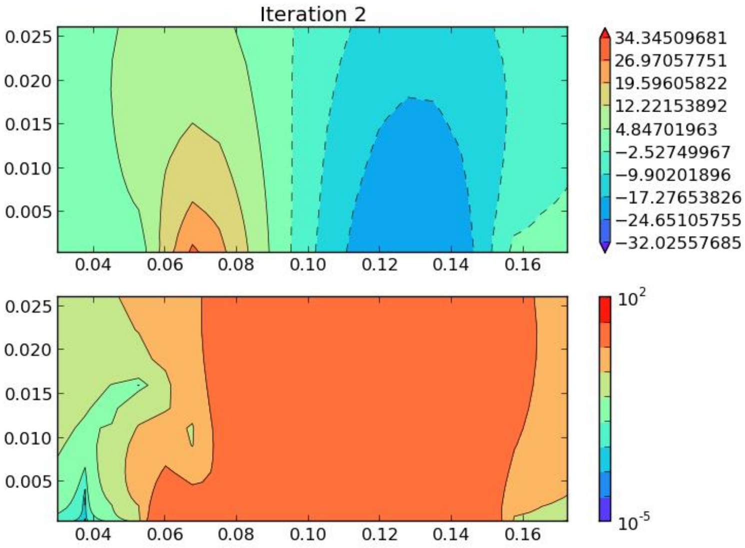

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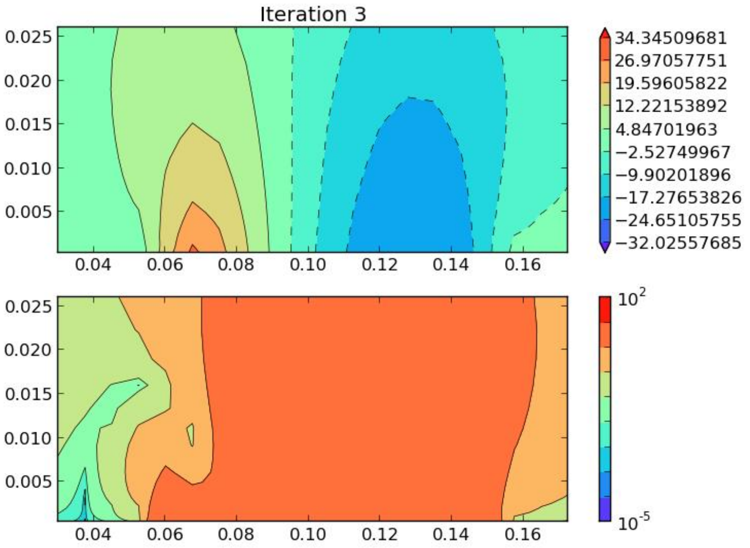

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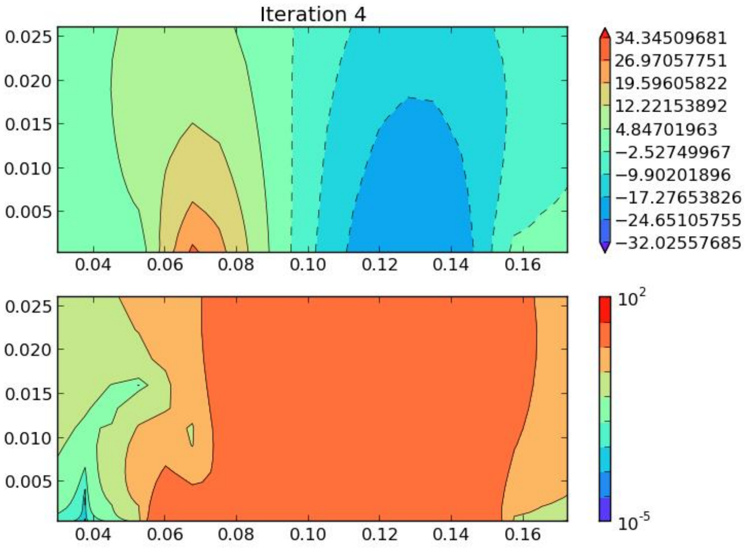

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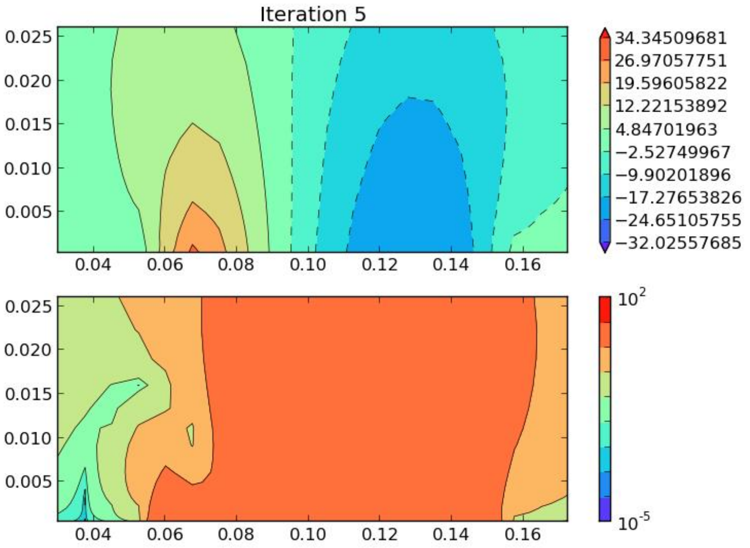

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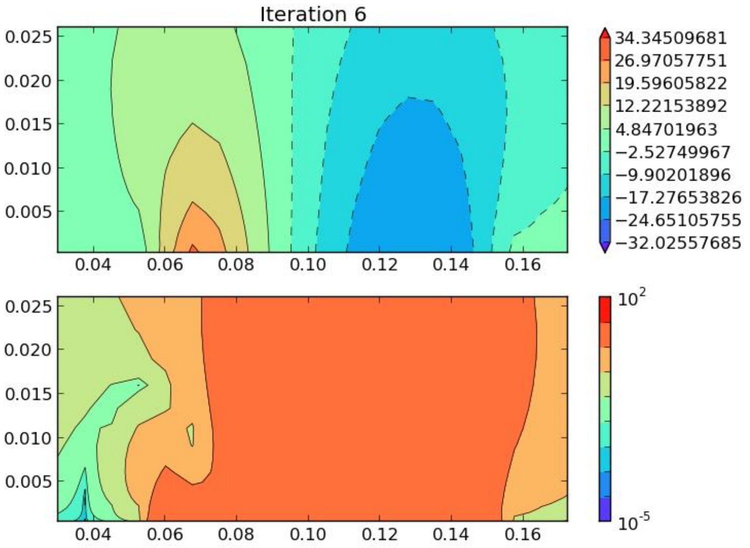

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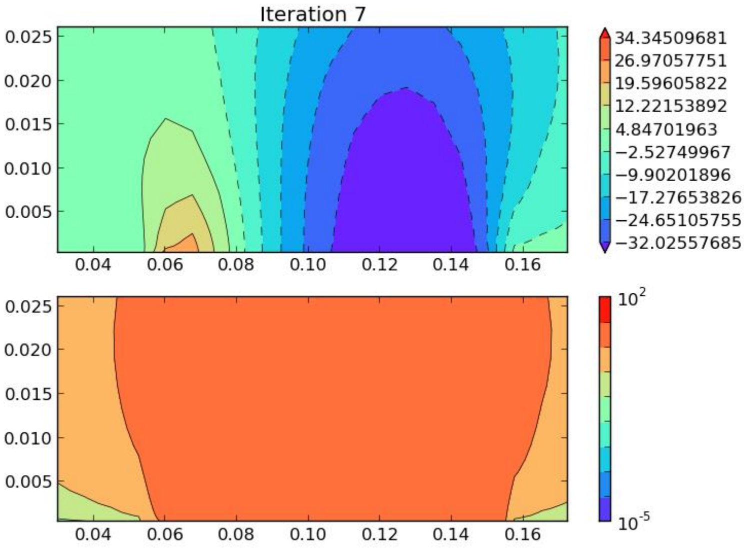

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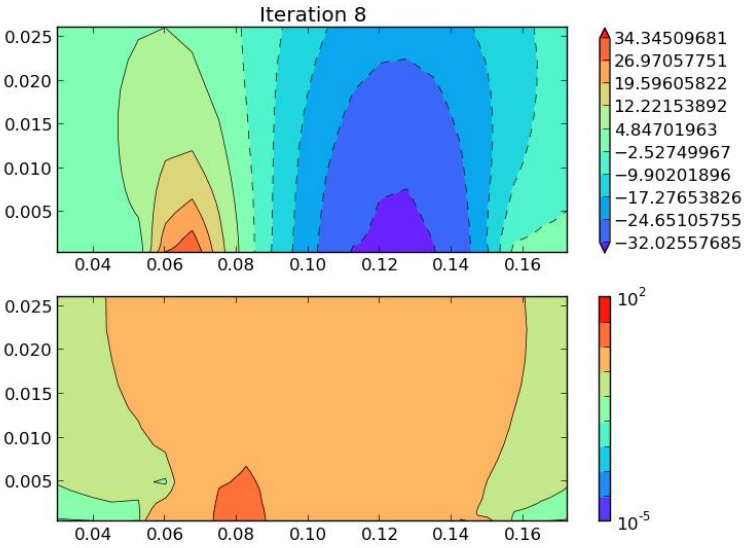
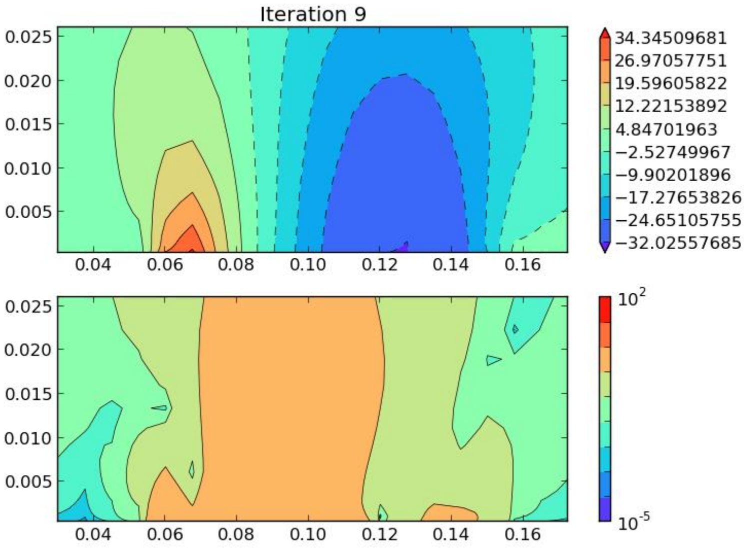
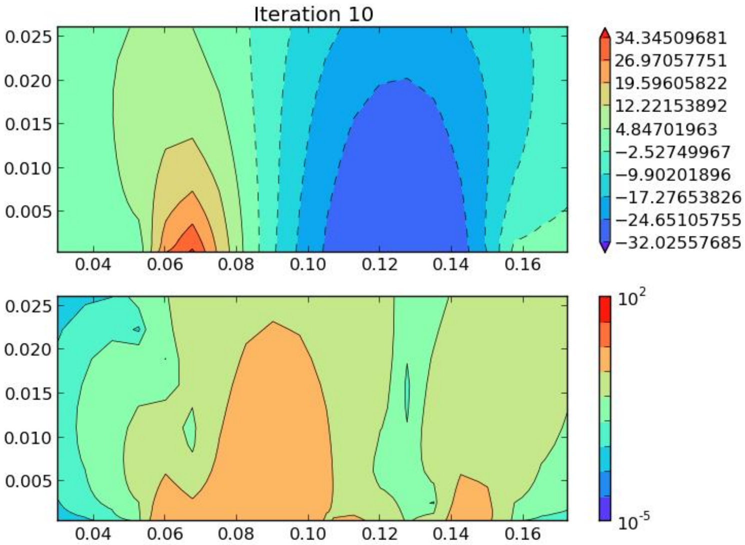


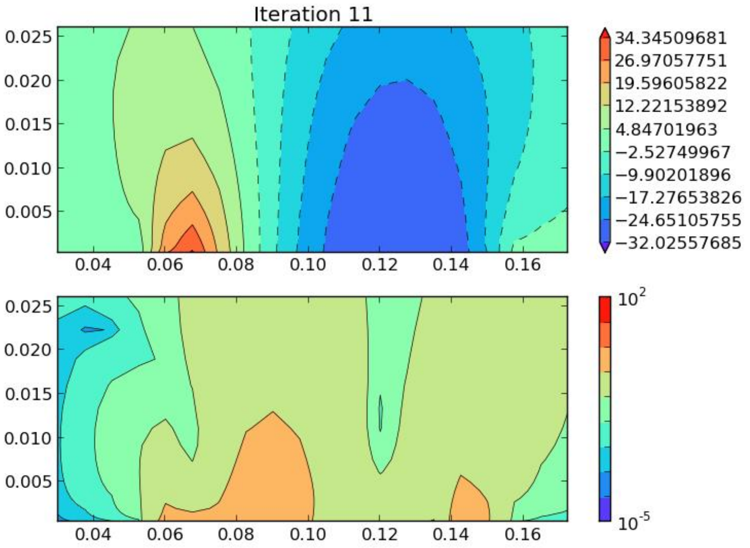
Convergence toward the measures, noise 0%



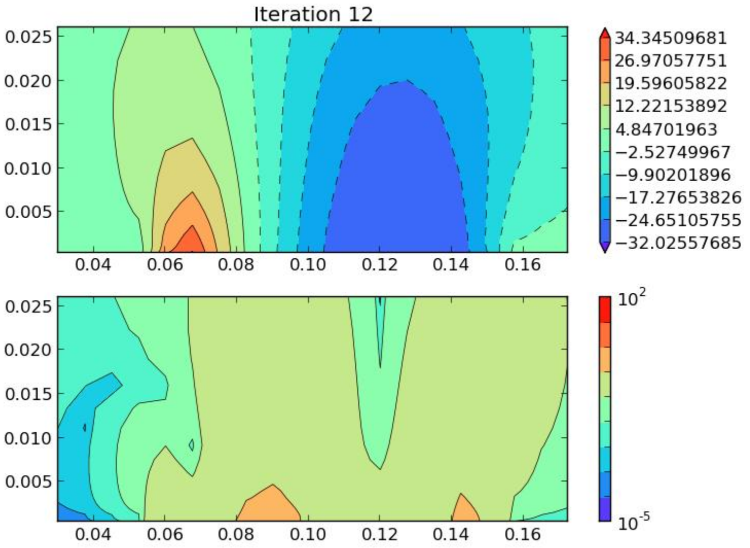
Convergence toward the measures, noise 0%



Convergence toward the measures, noise 0%



Convergence toward the measures, noise 0%



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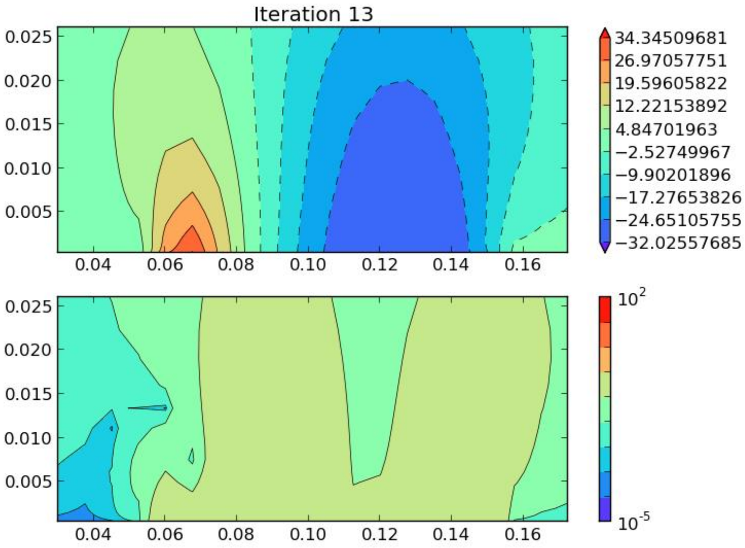

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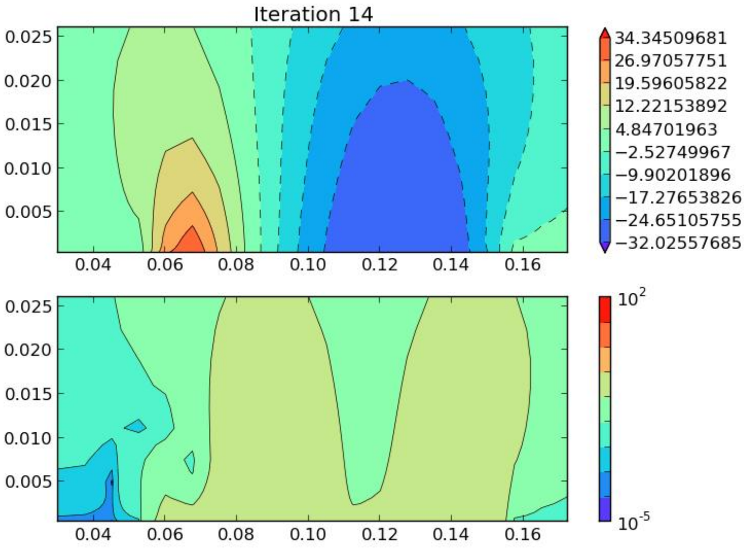

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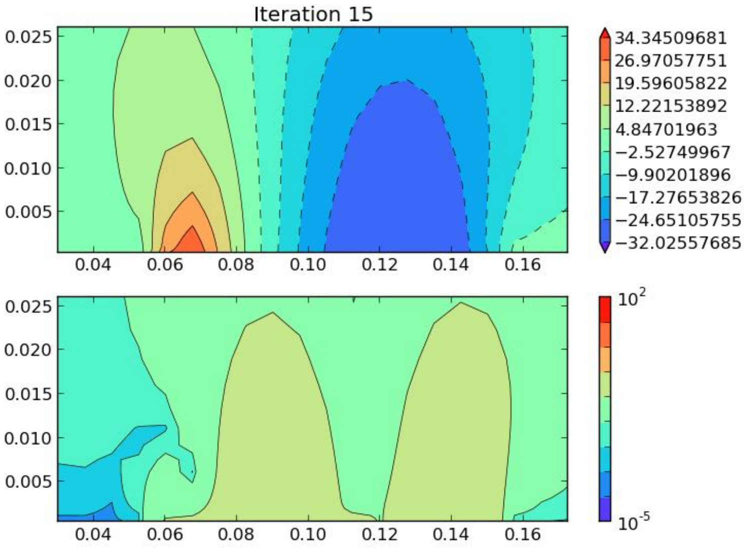

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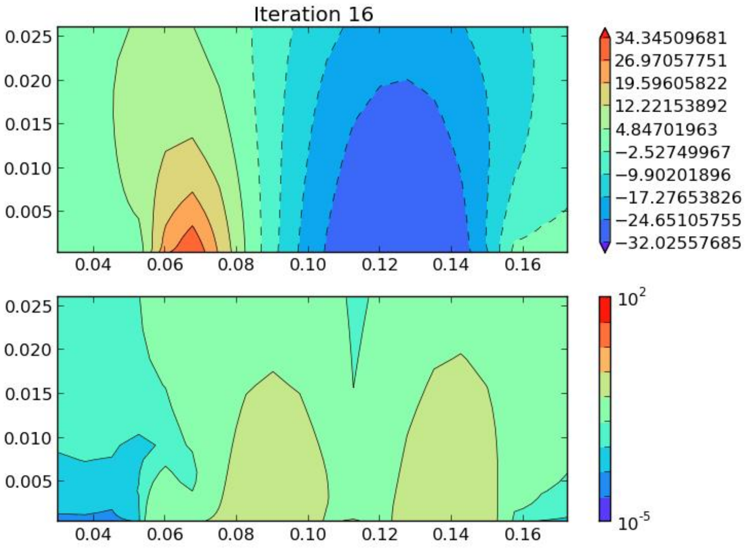

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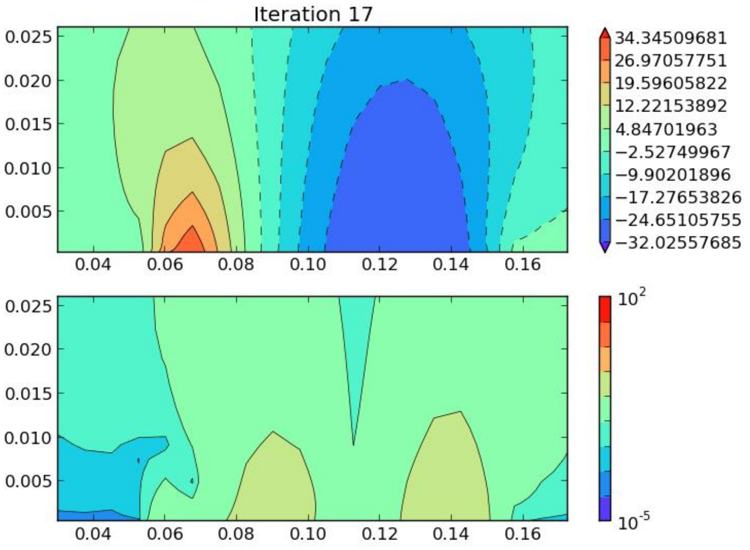
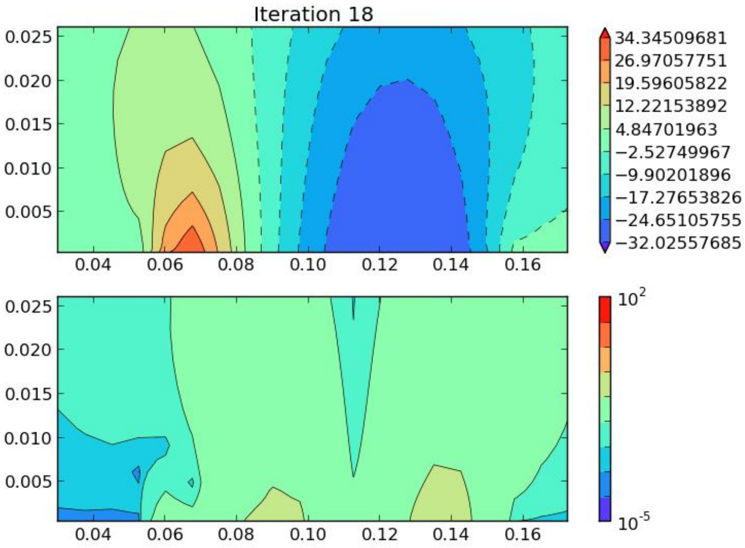


Convergence toward the measures, noise 0%



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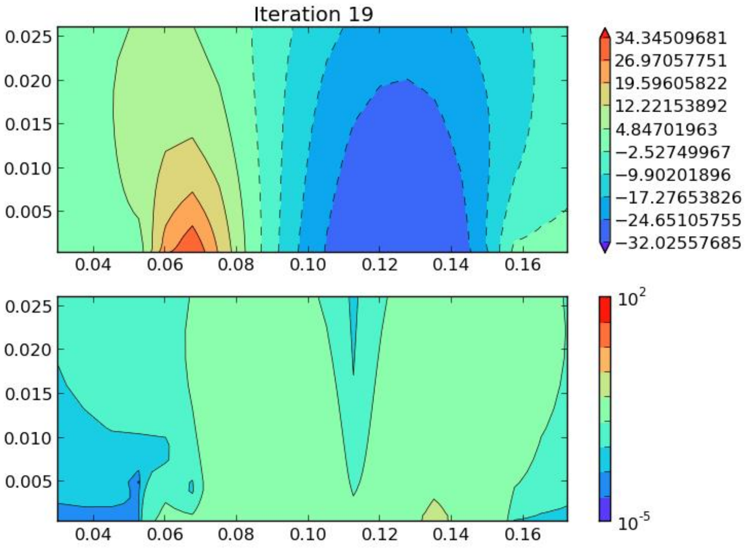

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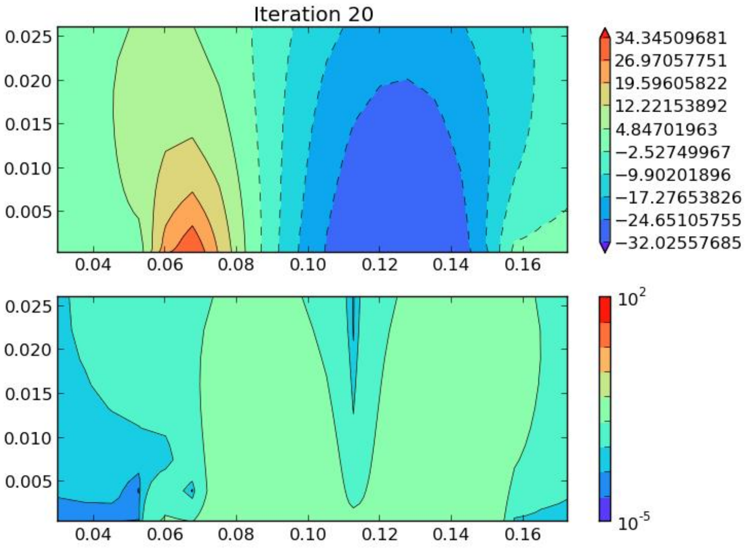

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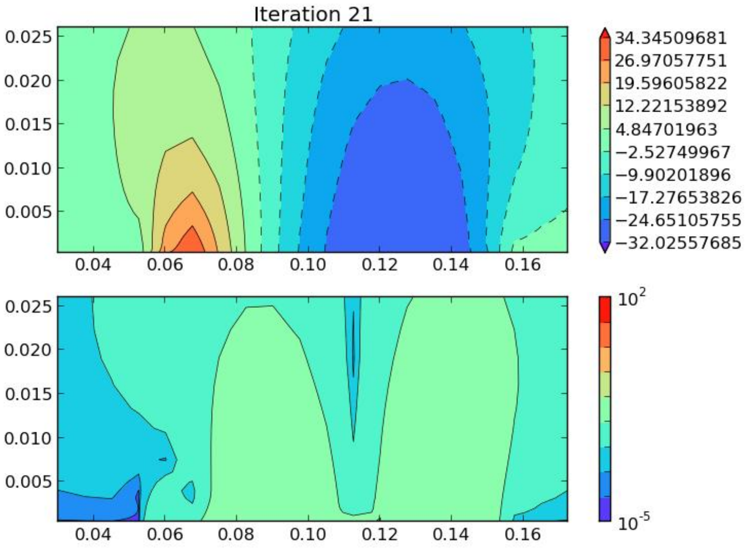

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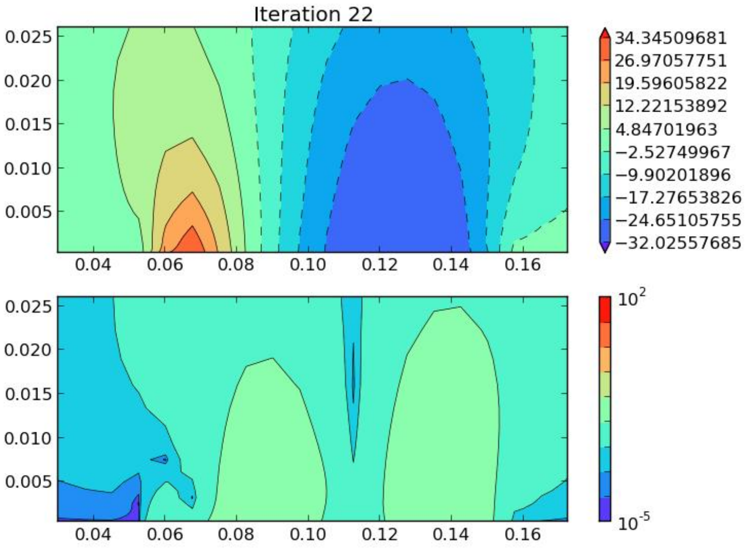

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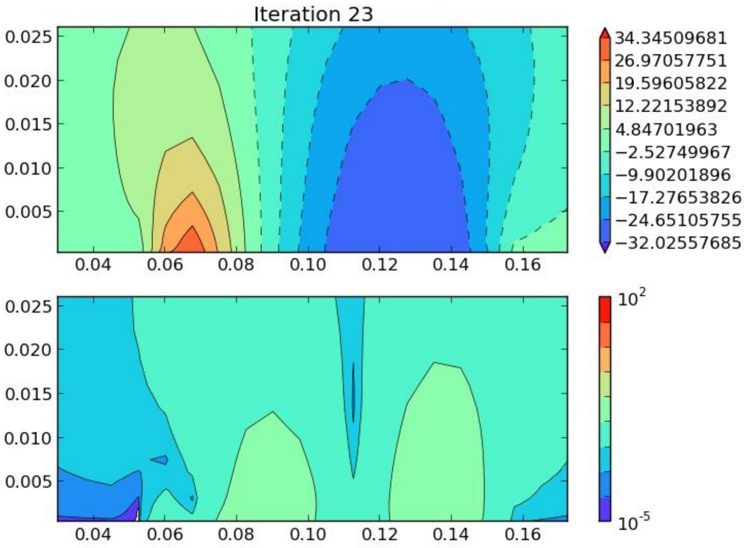

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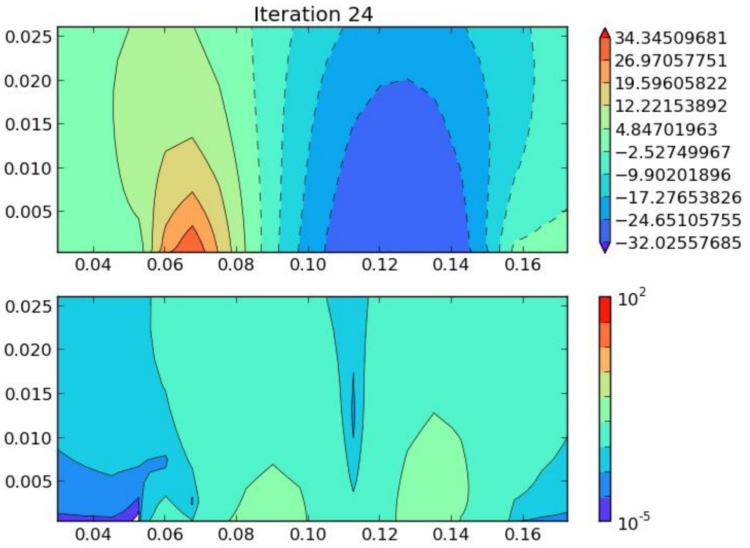

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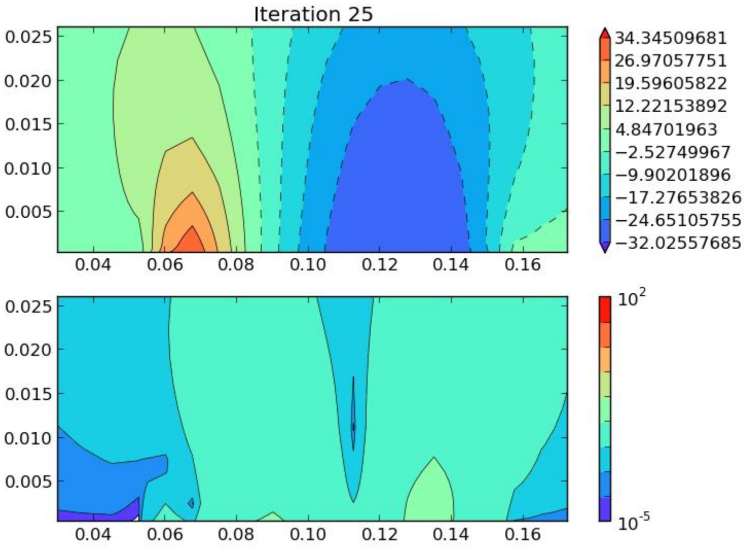

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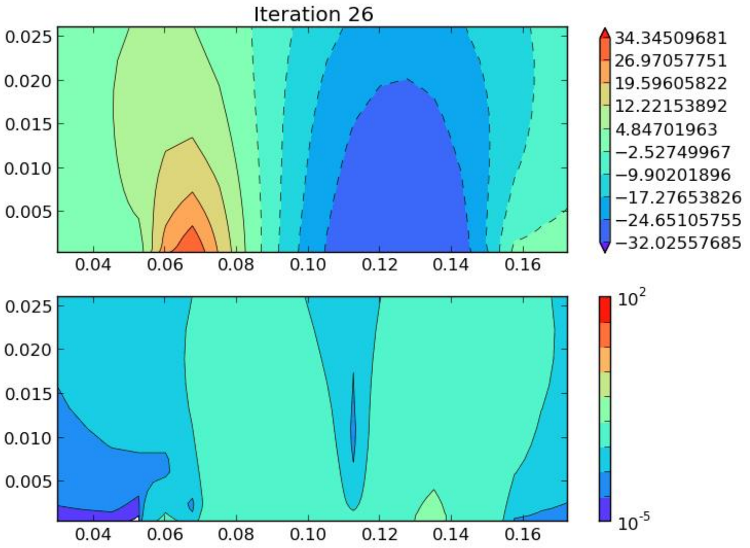

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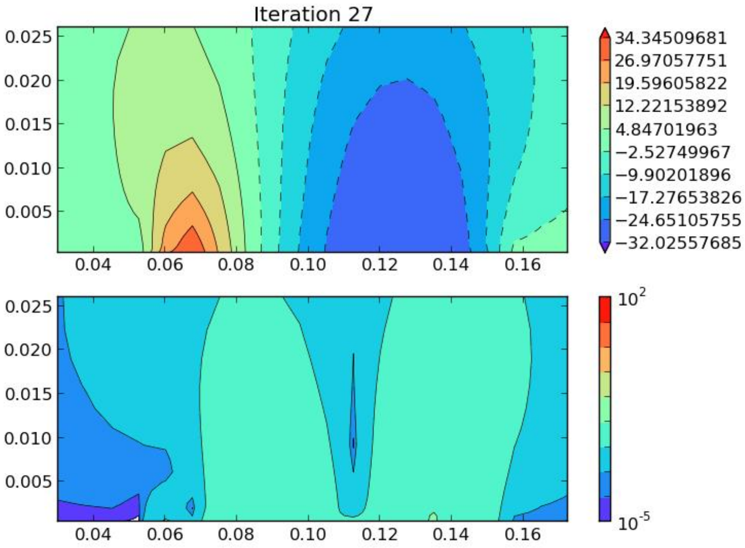

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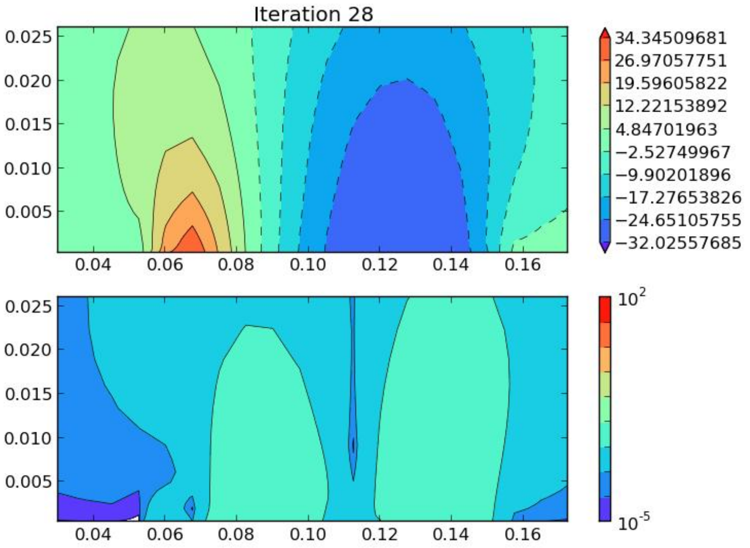

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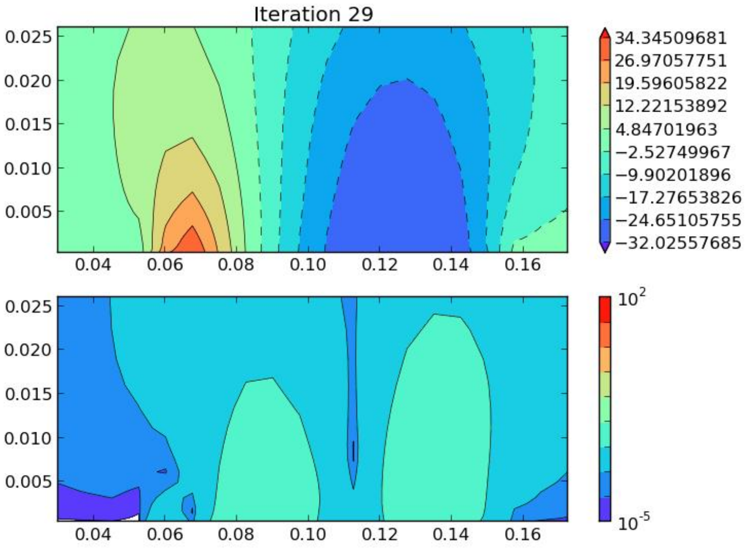

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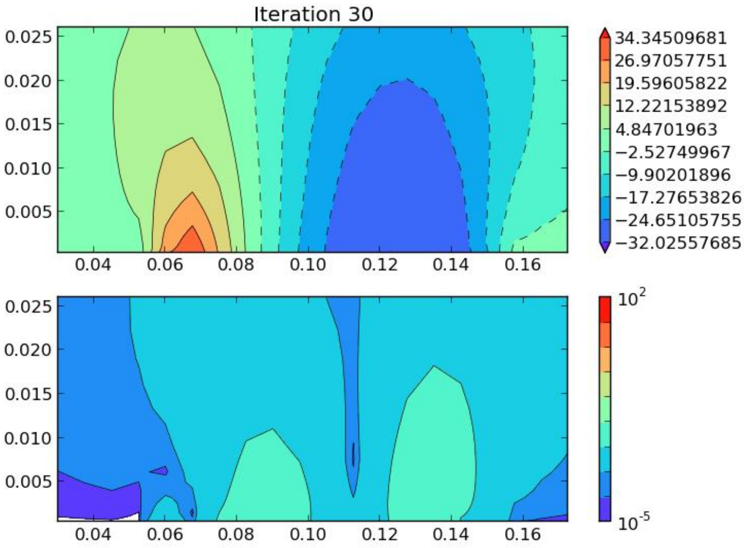

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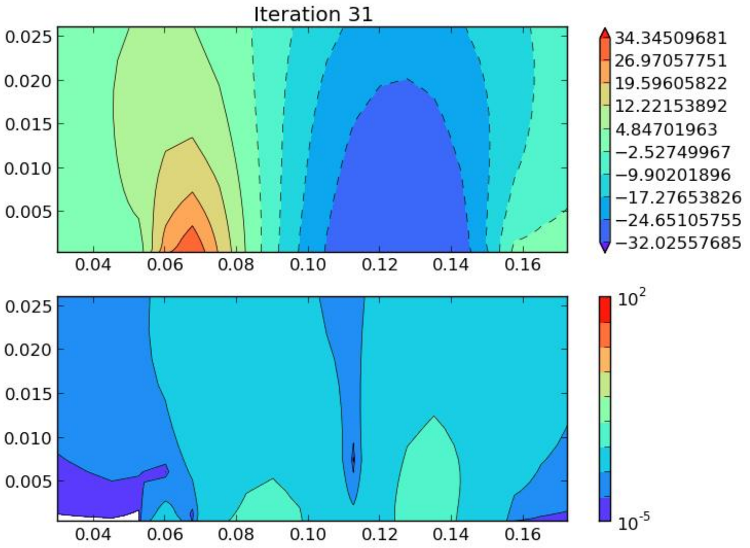

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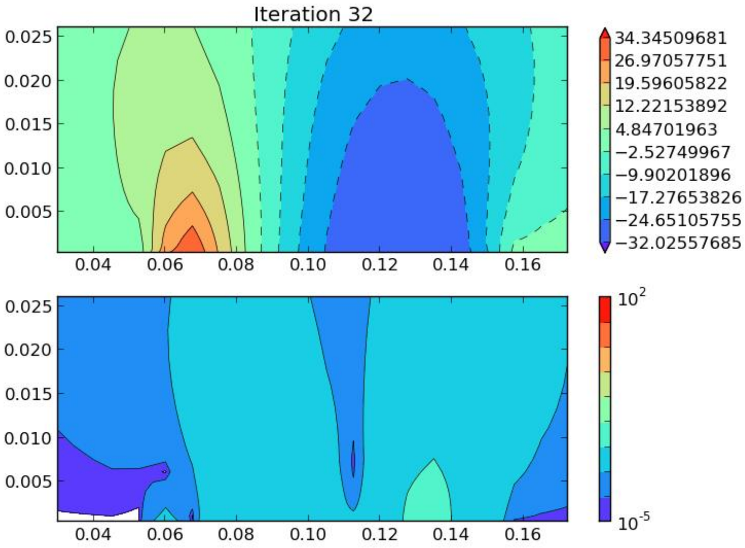

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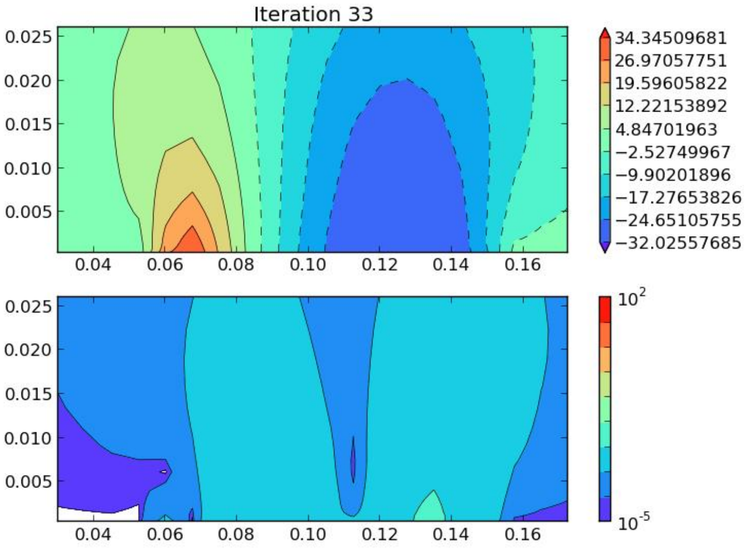

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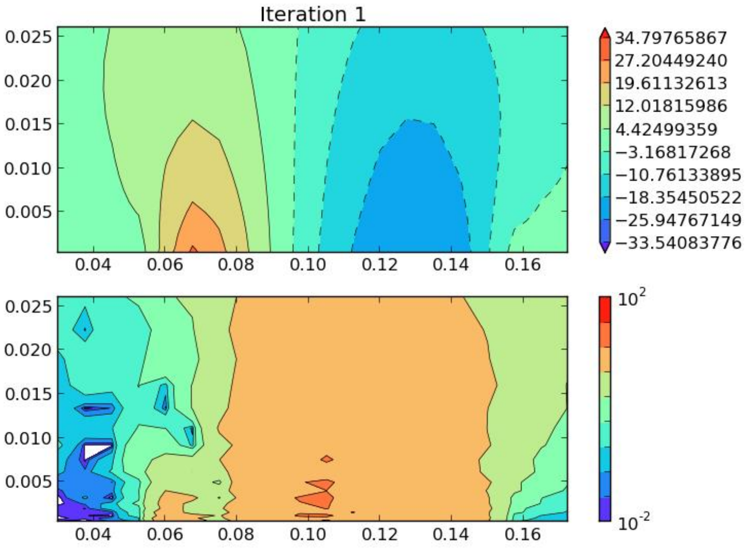

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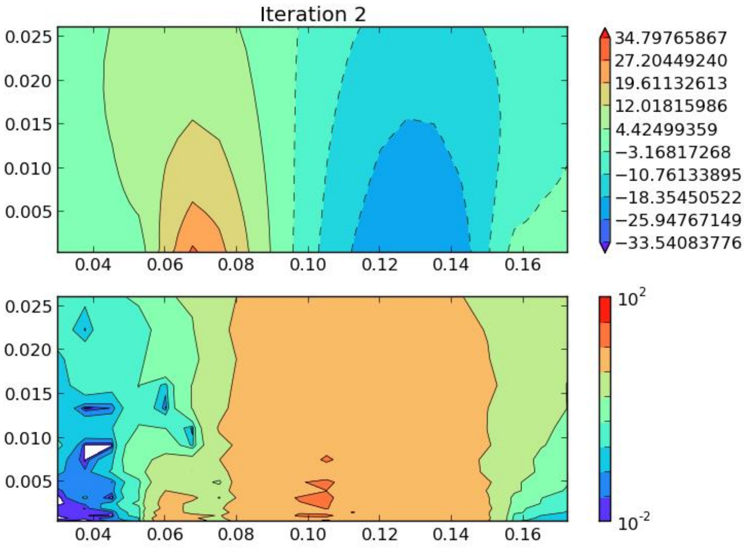

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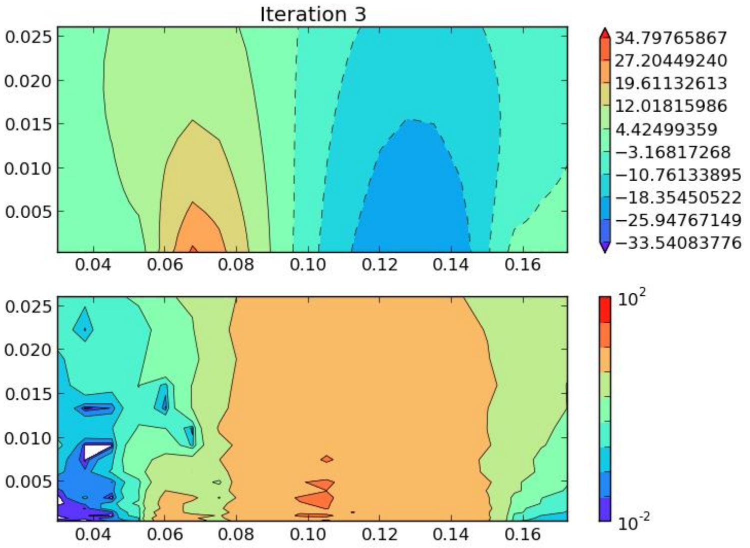

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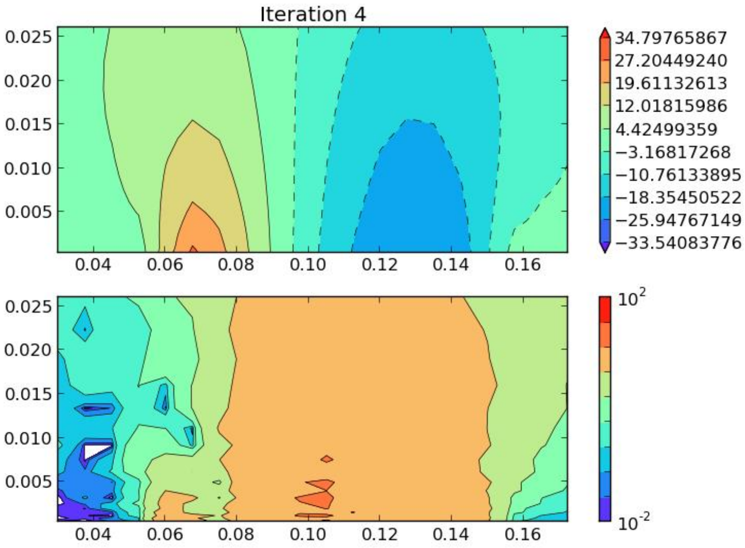

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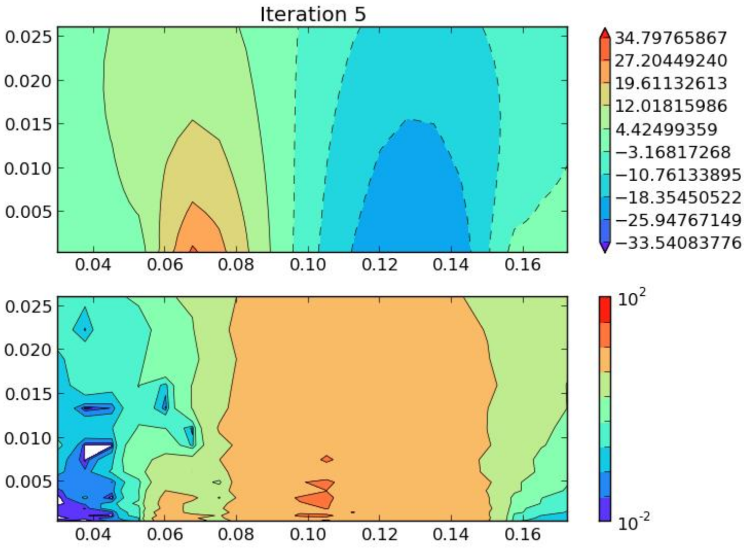

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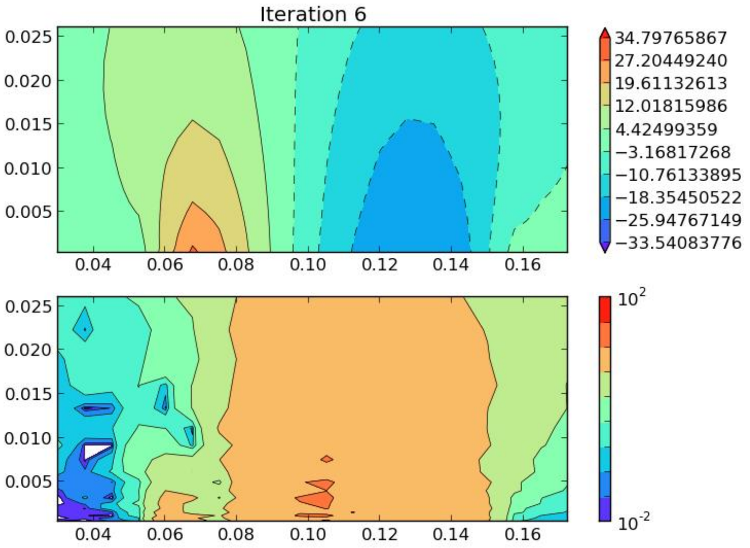

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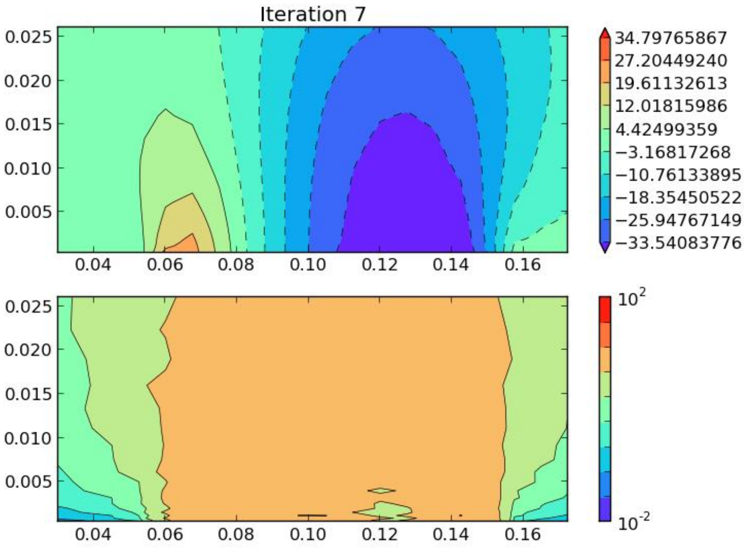
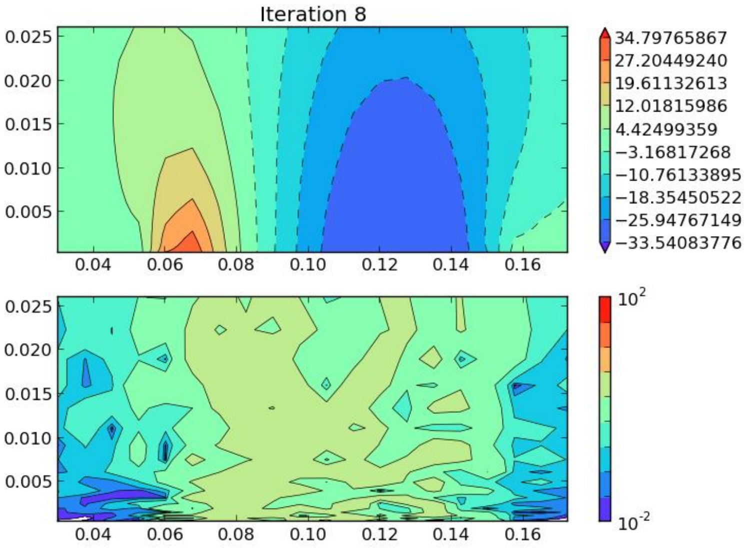


Convergence toward the measures, noise = 10%



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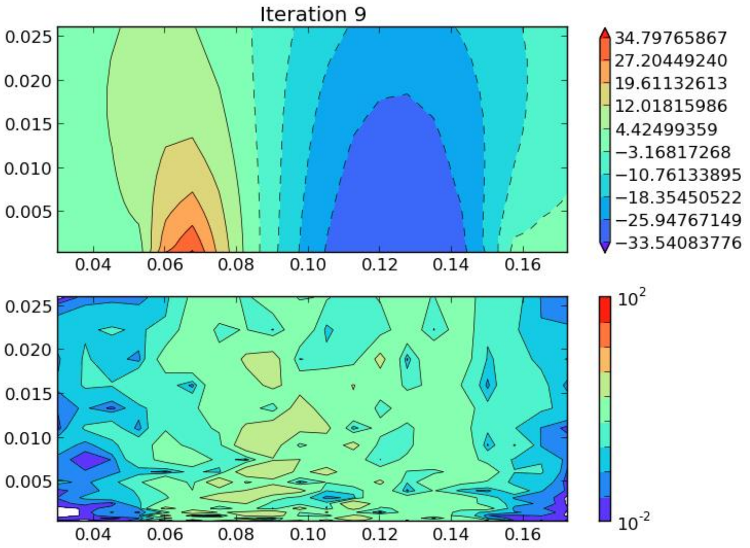

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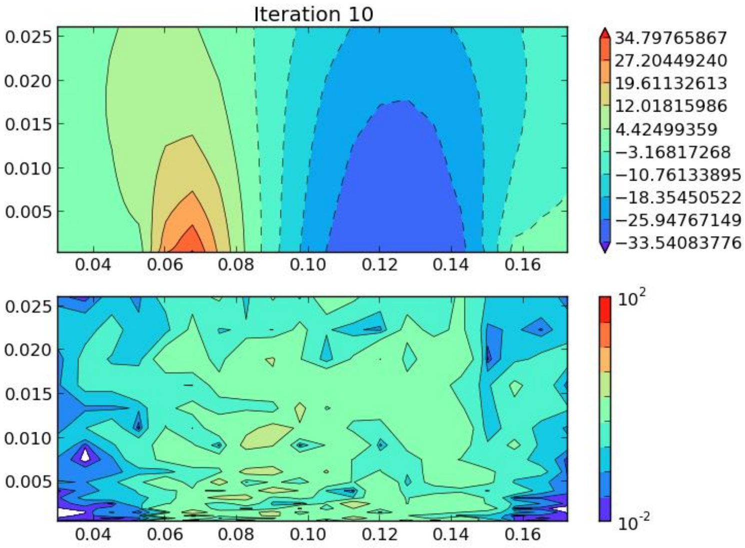

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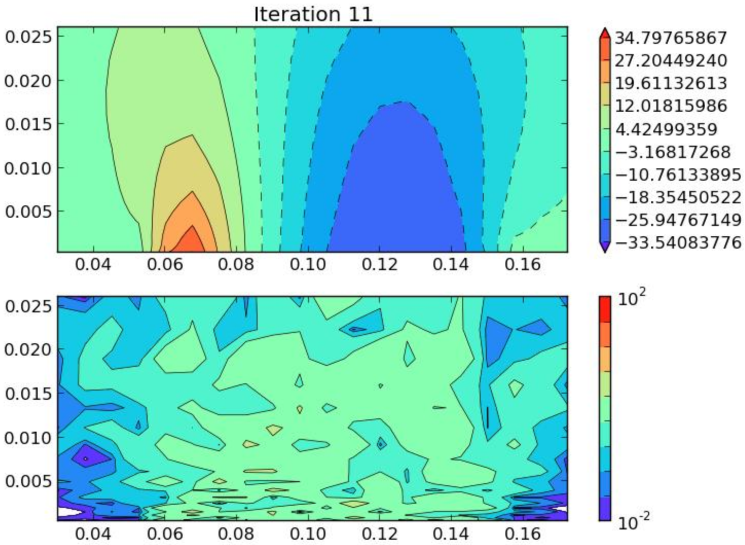

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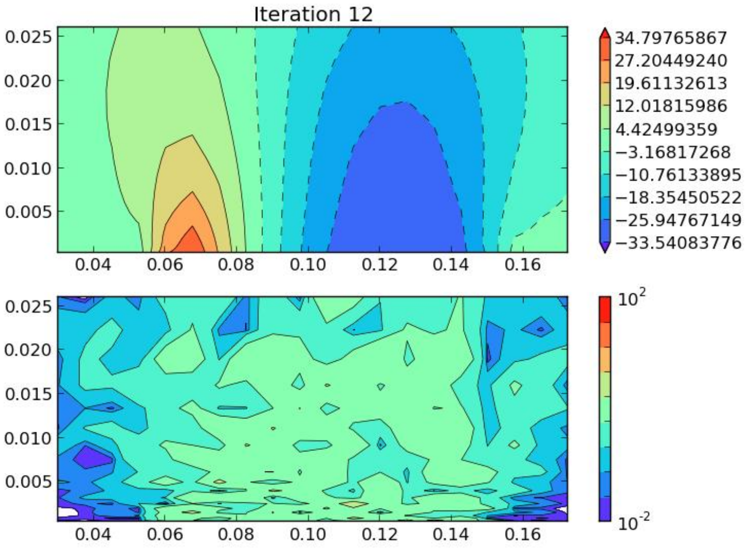

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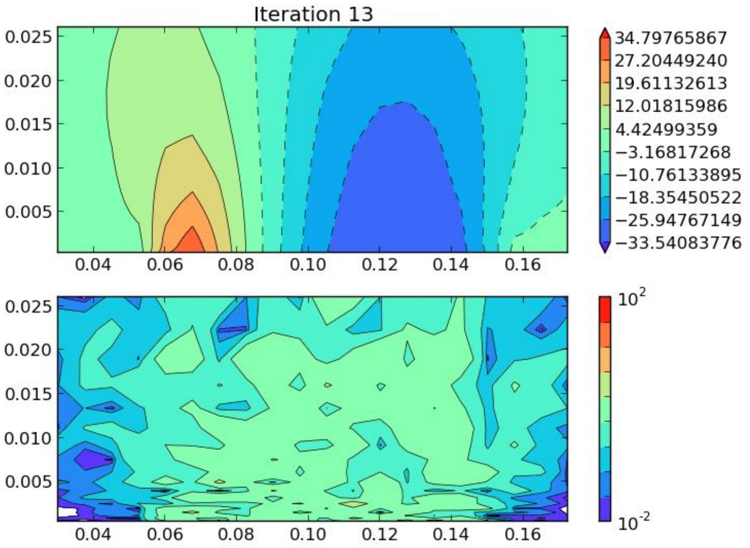
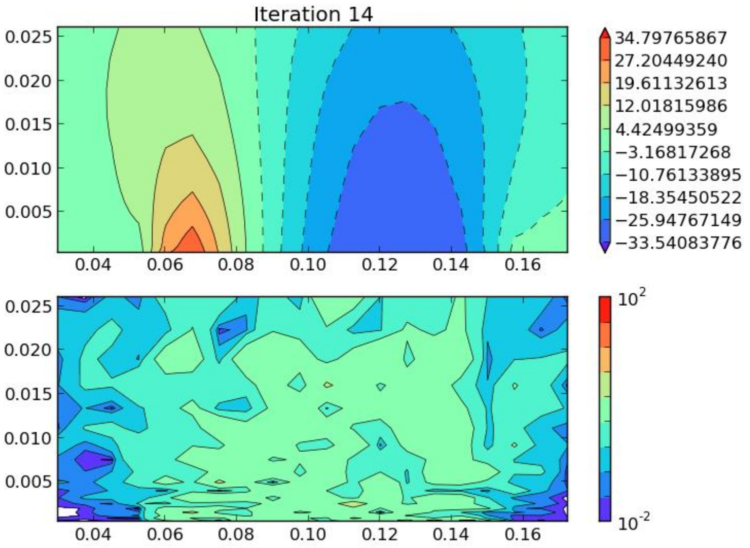


Convergence toward the measures, noise = 10%



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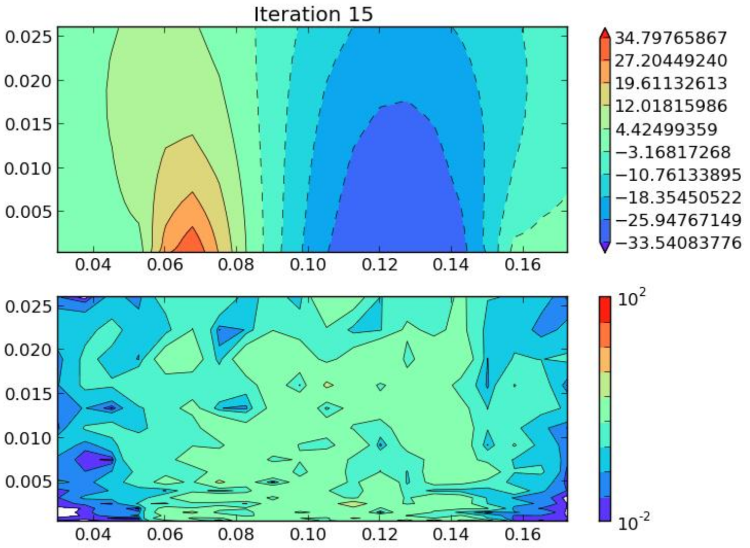

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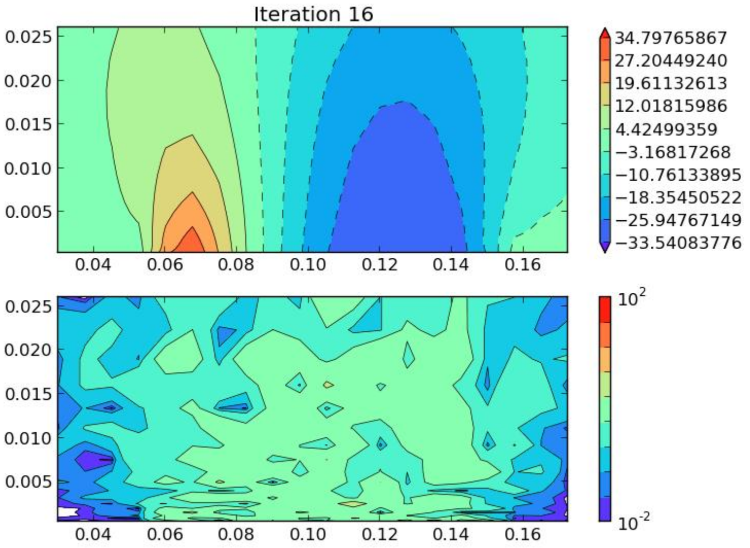
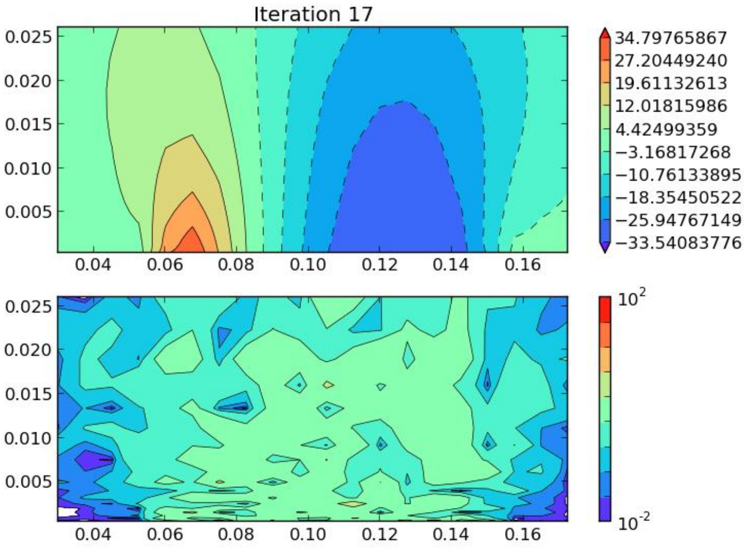


Convergence toward the measures, noise = 10%



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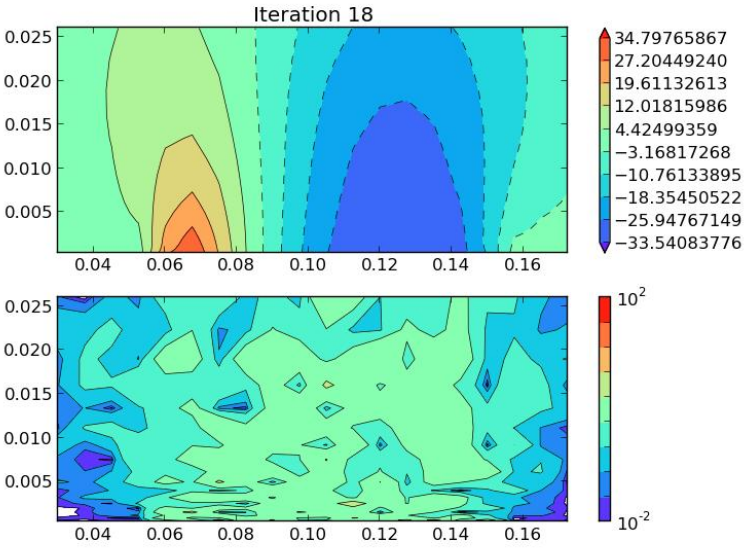

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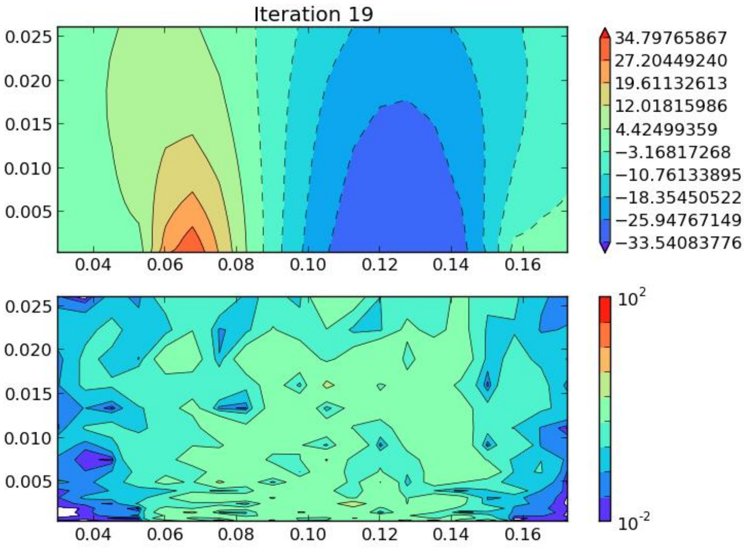
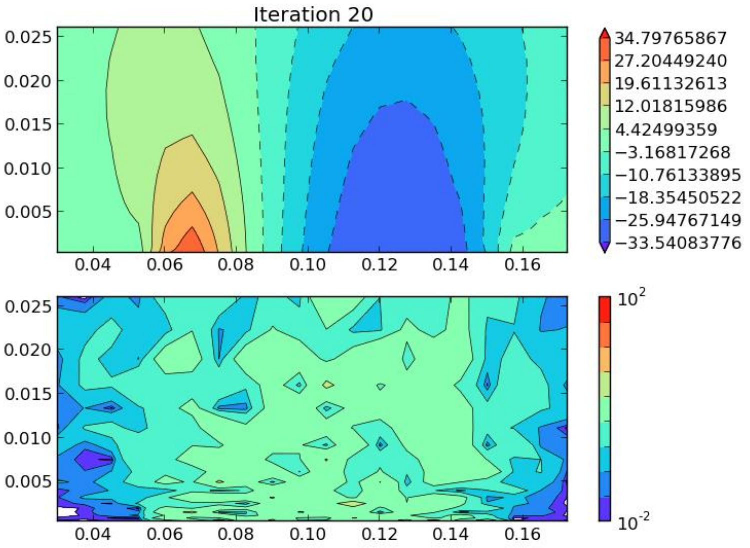
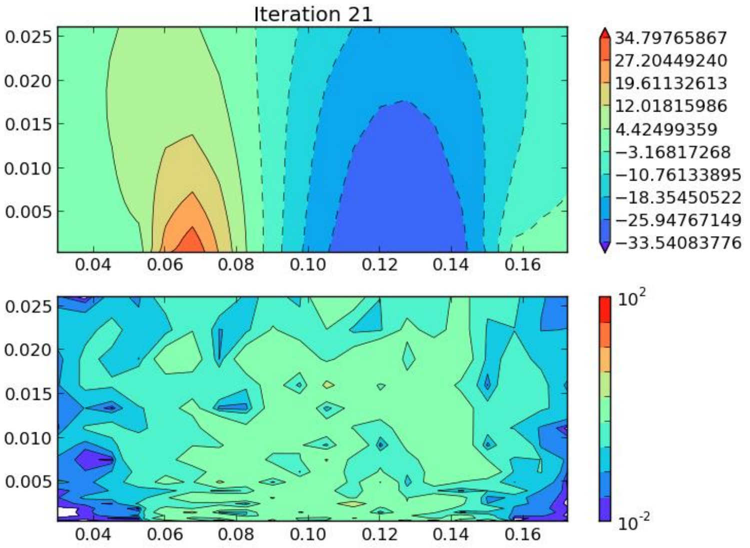


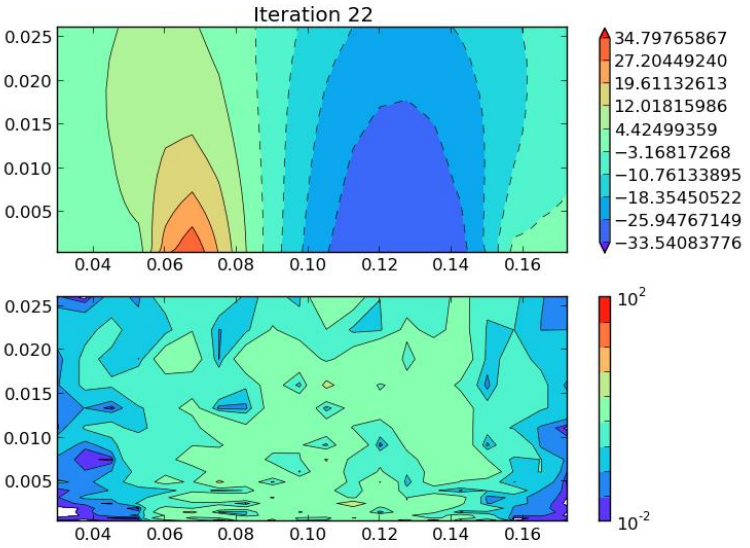
Convergence toward the measures, noise = 10%

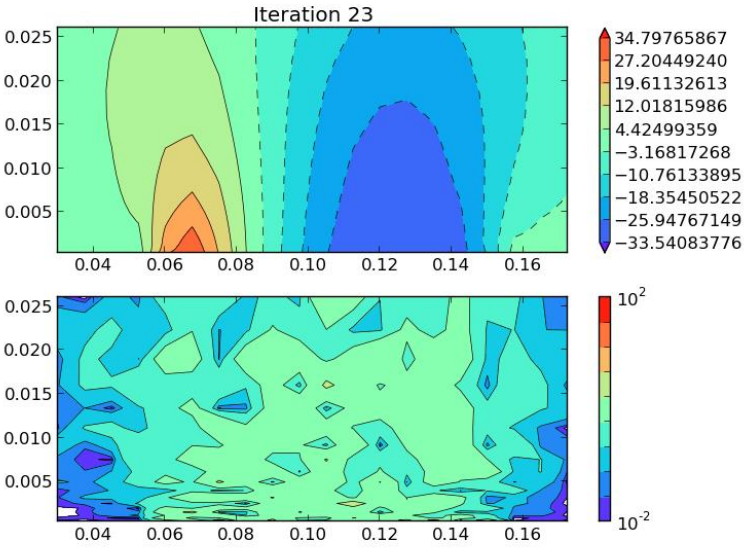


Convergence toward the measures, noise = 10%

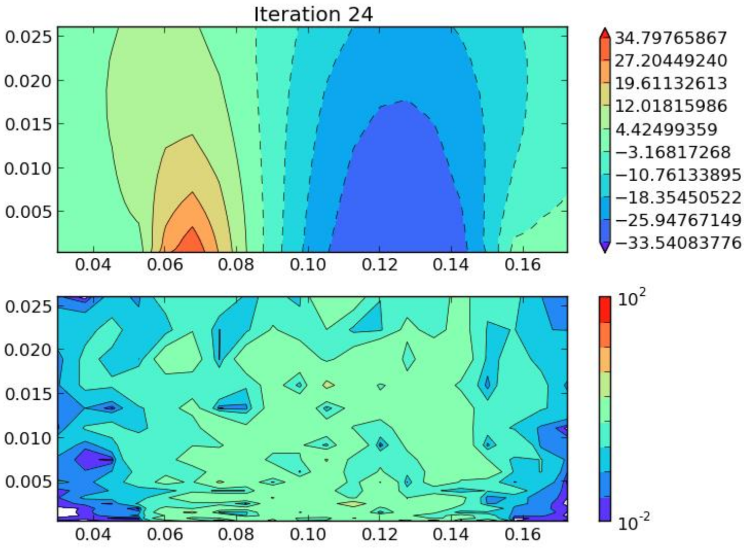


Convergence toward the measures, noise = 10%

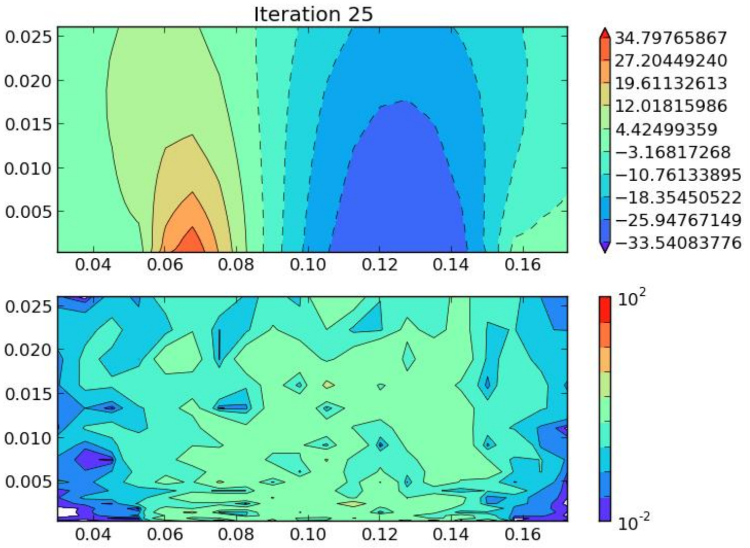



Iteration 23
0.025
0.020
0.015
0.010
0.005
0.04
0.06
0.08
0.10
0.12
0.14
0.16
34.79765867
27.20449240
19.61132613
12.01815986
4.42499359
−3.16817268
−10.76133895
−18.35450522
−25.94767149
−33.54083776
10^2
10^-2

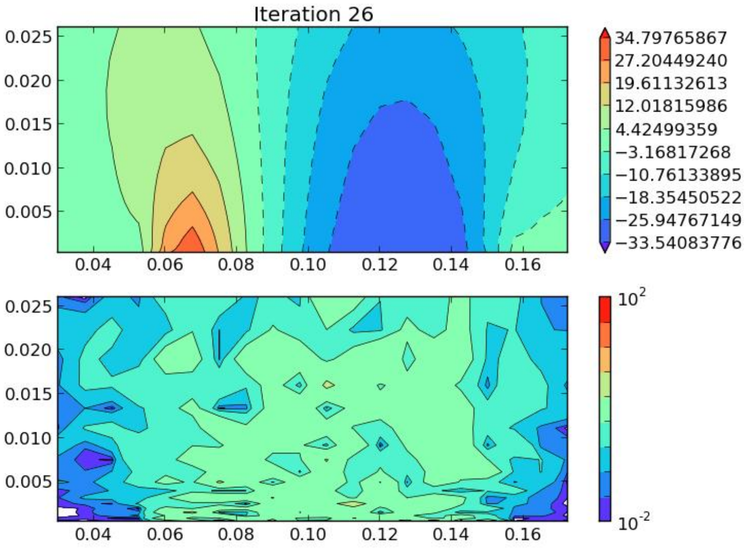
Convergence toward the measures, noise = 10%



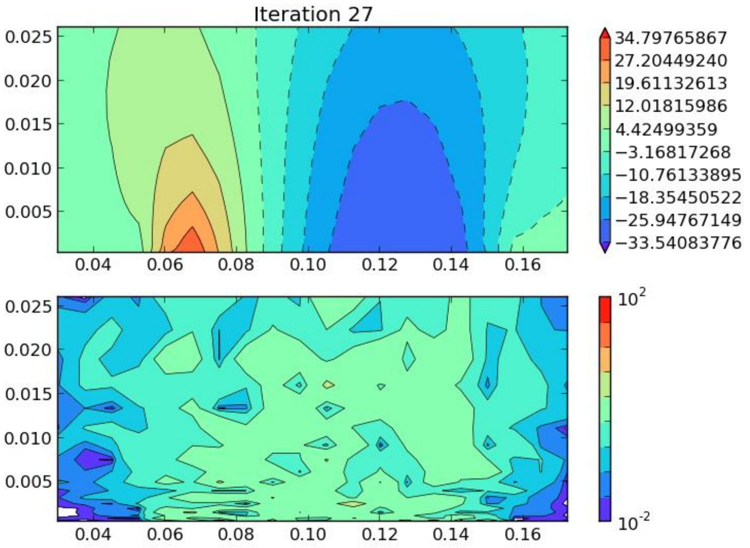
Convergence toward the measures, noise = 10%



Convergence toward the measures, noise = 10%



Convergence toward the measures, noise = 10%



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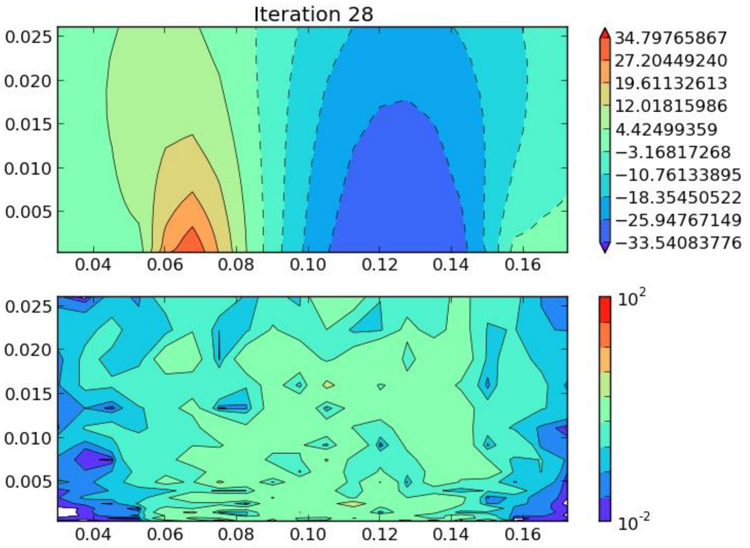
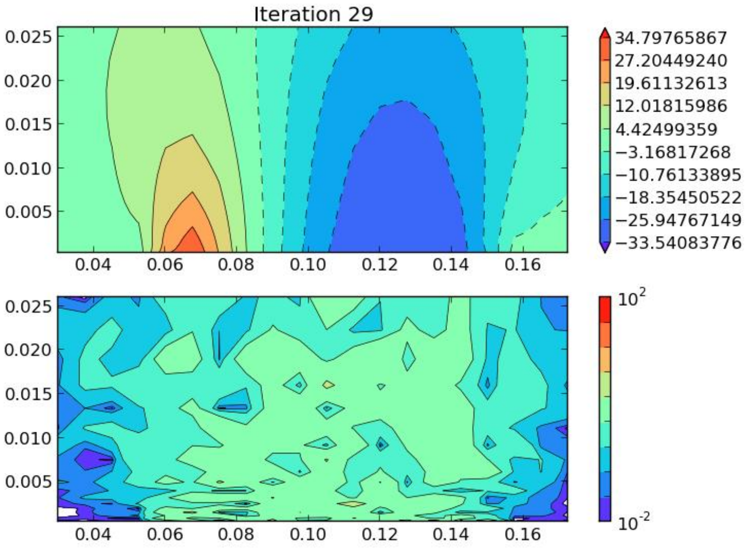
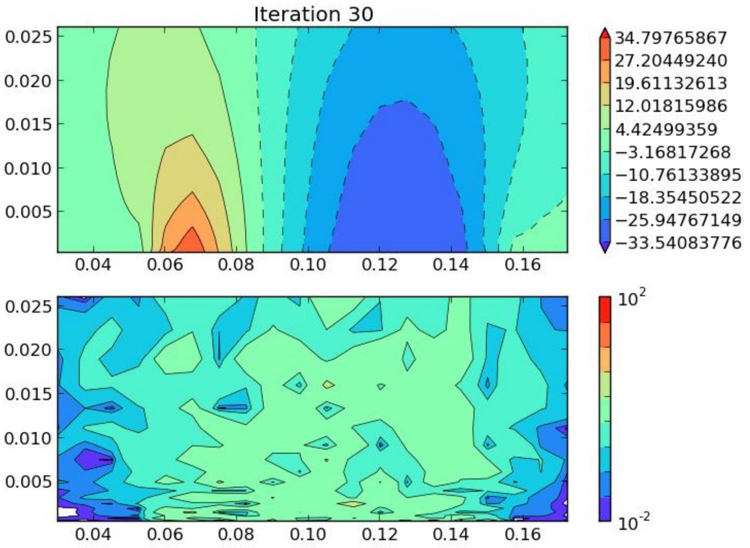


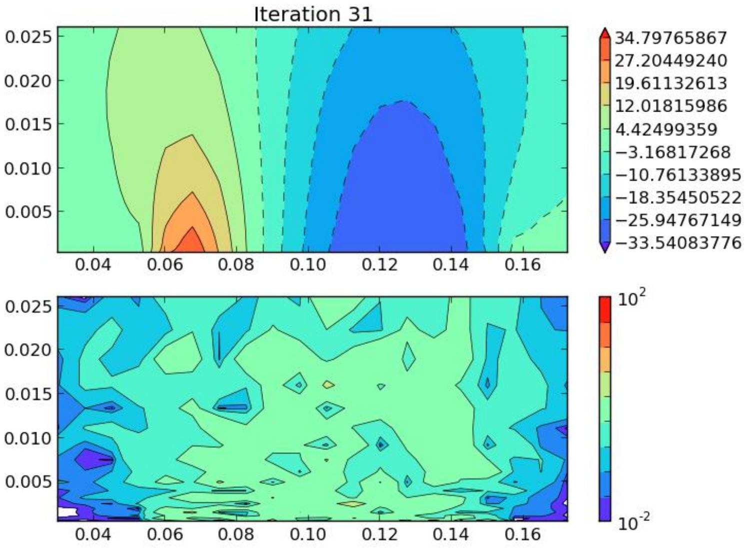
Convergence toward the measures, noise = 10%



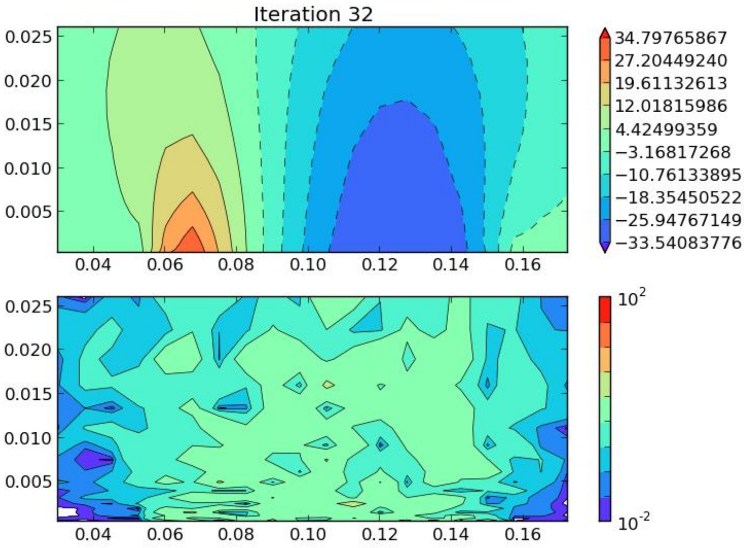
Convergence toward the measures, noise = 10%



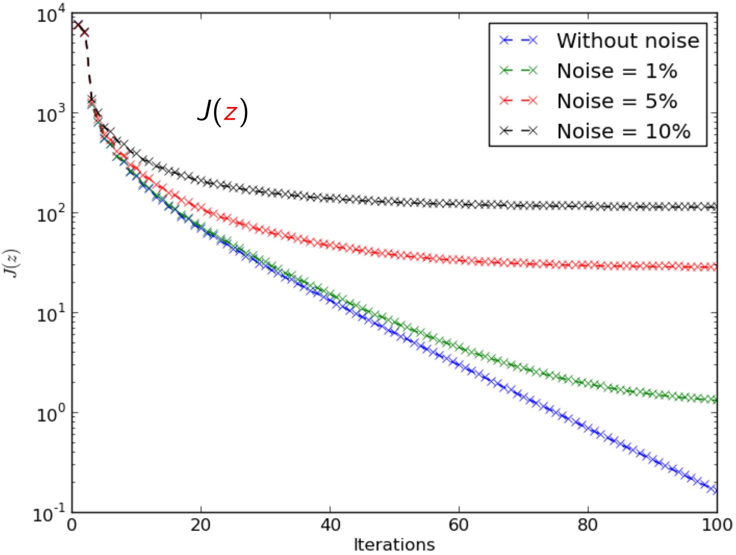
Convergence toward the measures, noise = 10%



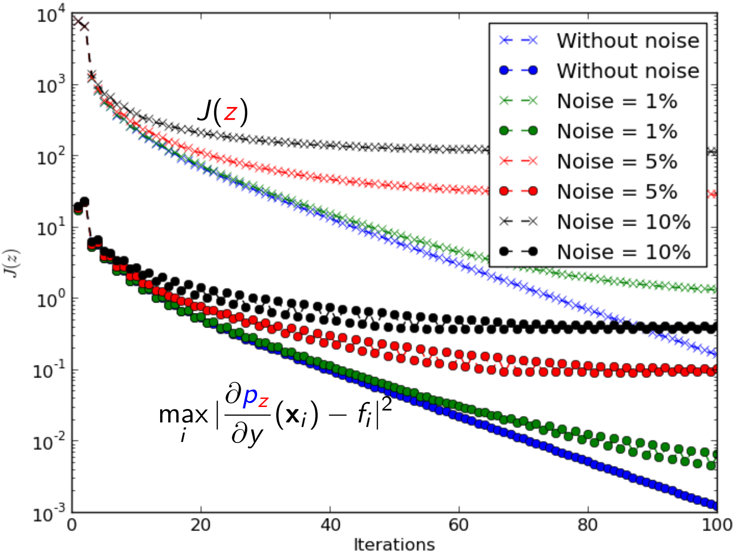
Convergence toward the measures, noise = 10%



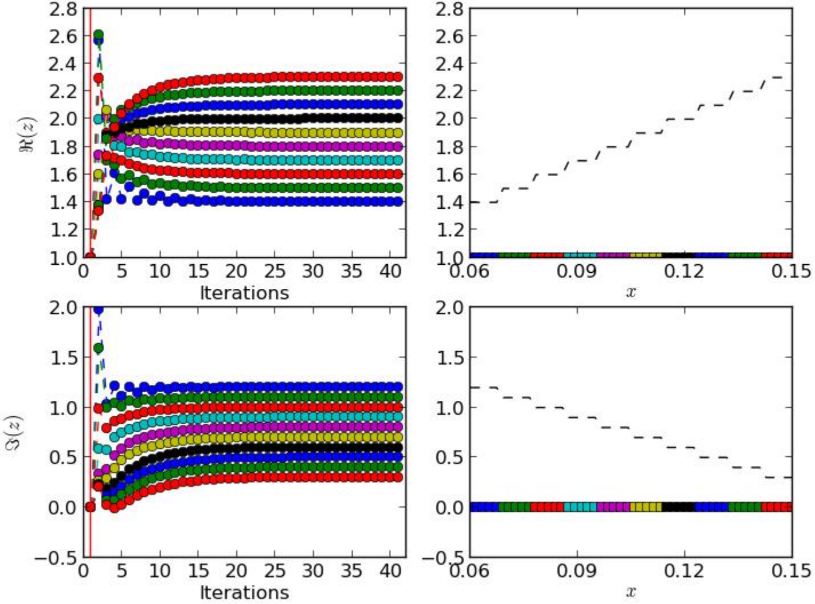
Evolution of the functional



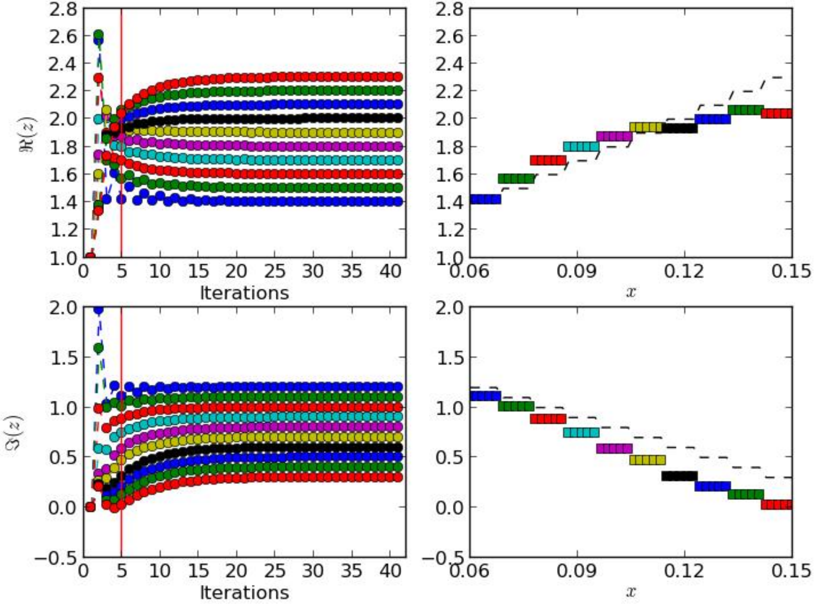
Evolution of the functional



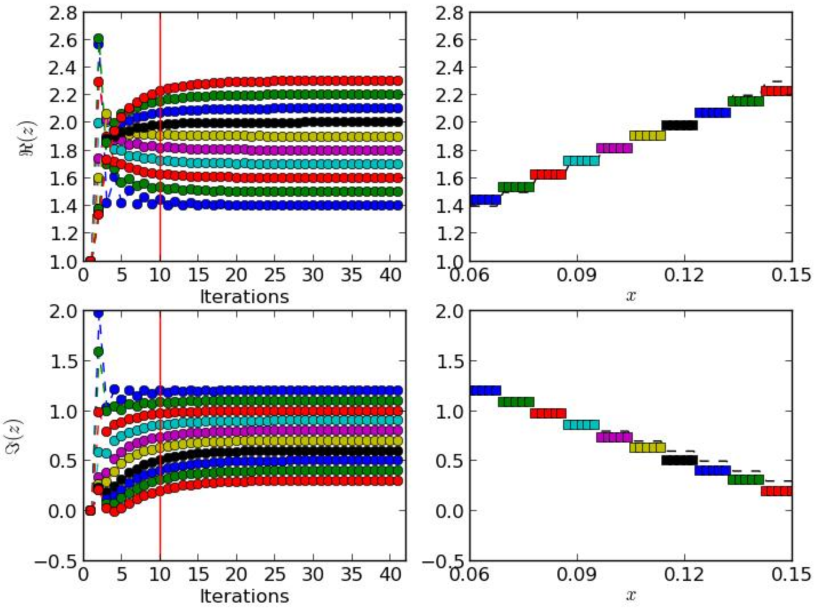
Variable impedance, Iteration 1, noise 0%



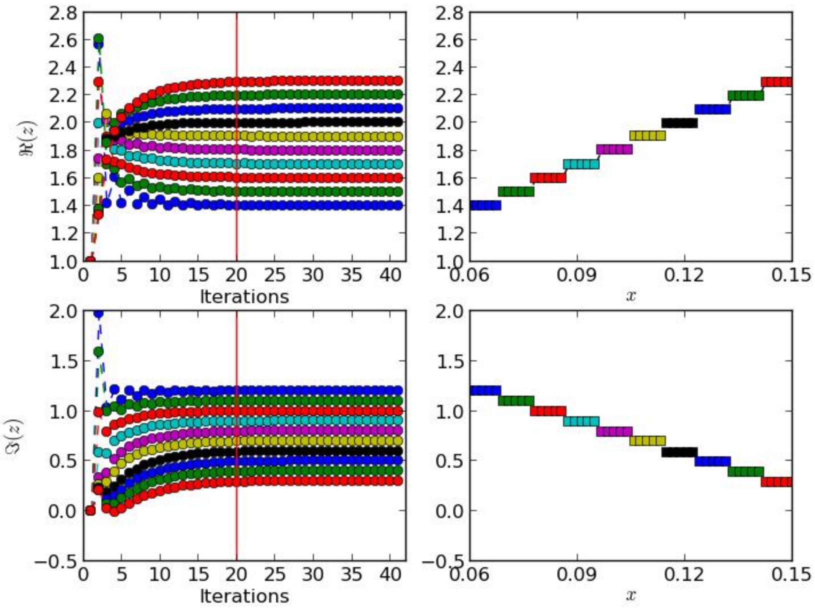
Variable impedance, Iteration 5, noise 0%



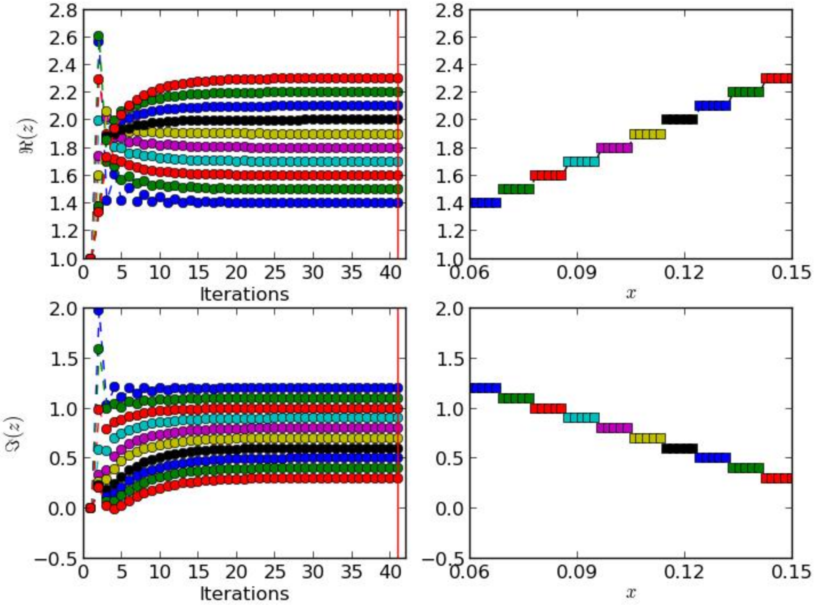
Variable impedance, Iteration 10, noise 0%



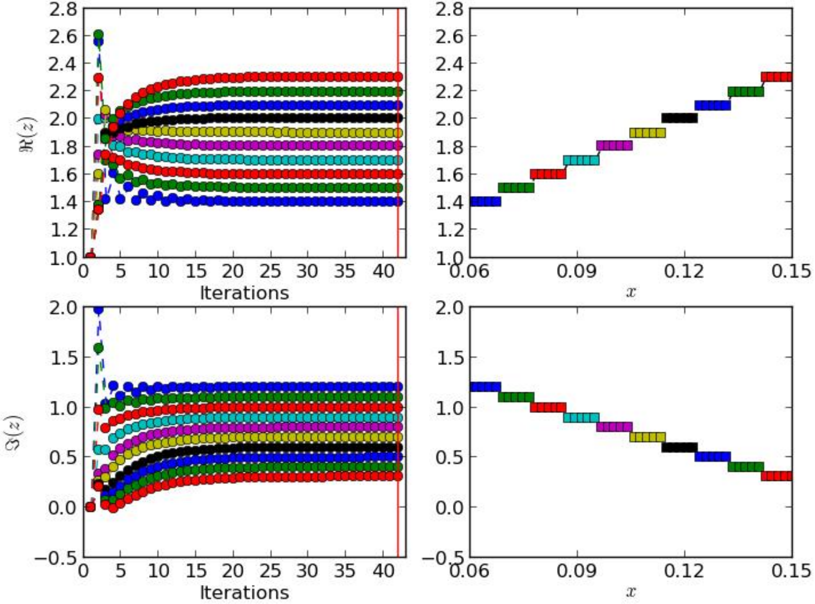
Variable impedance, Iteration 20, noise 0%



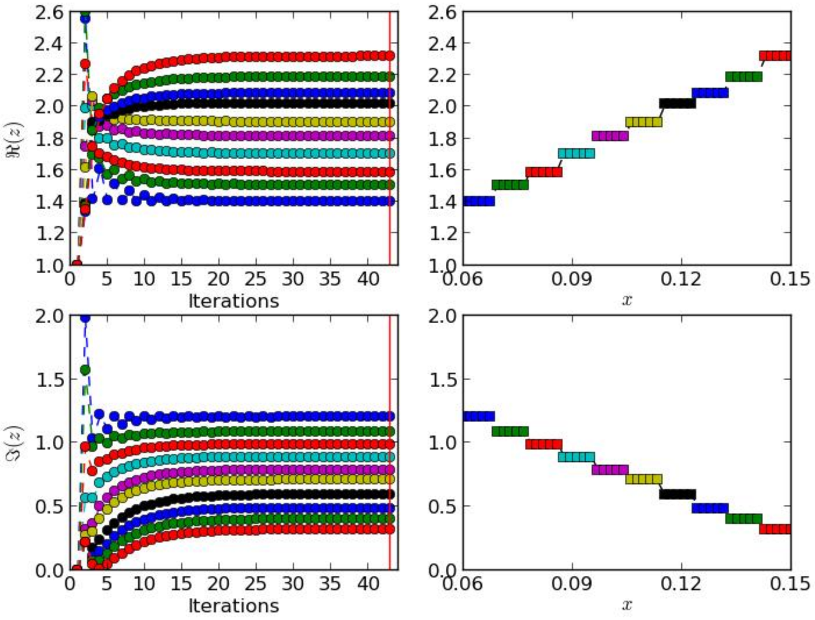
Variable impedance, Iteration 41, noise 0%



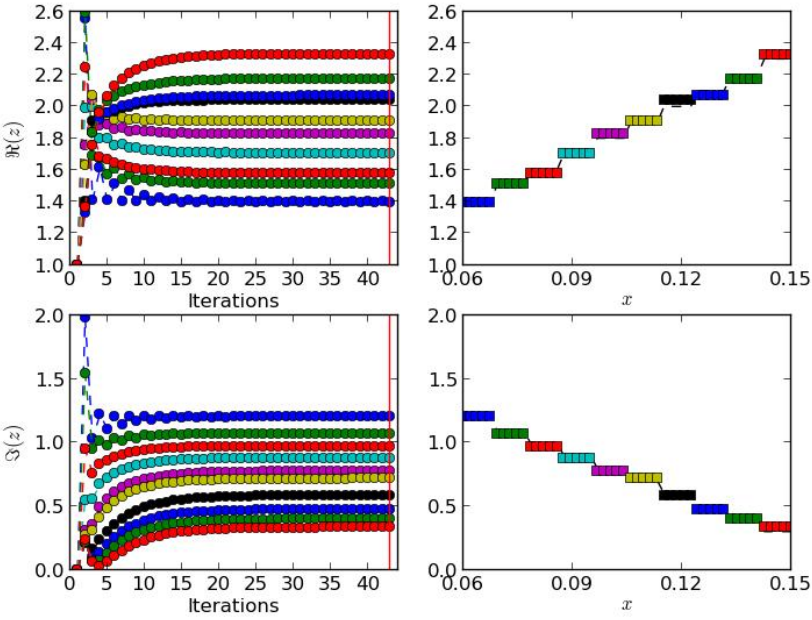
Variable impedance, iteration 43 and noise 1%



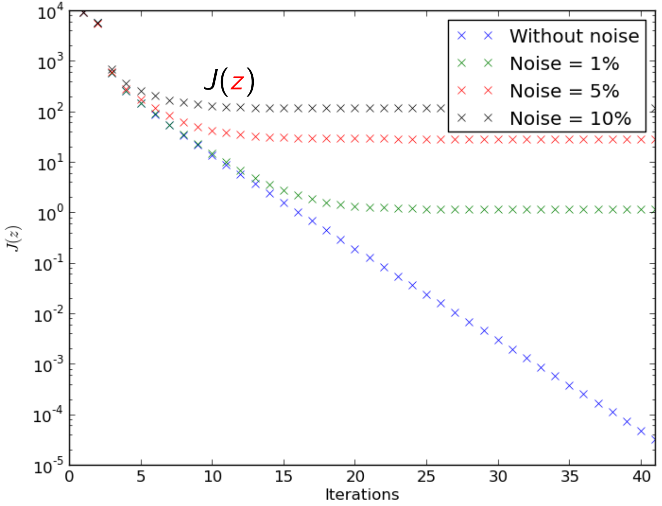
Variable impedance, iteration 43 and noise 5%



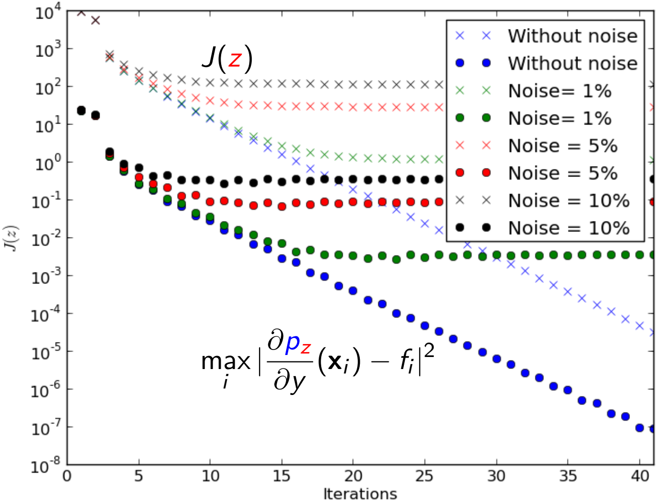
Variable impedance, iteration 43 and noise 10%



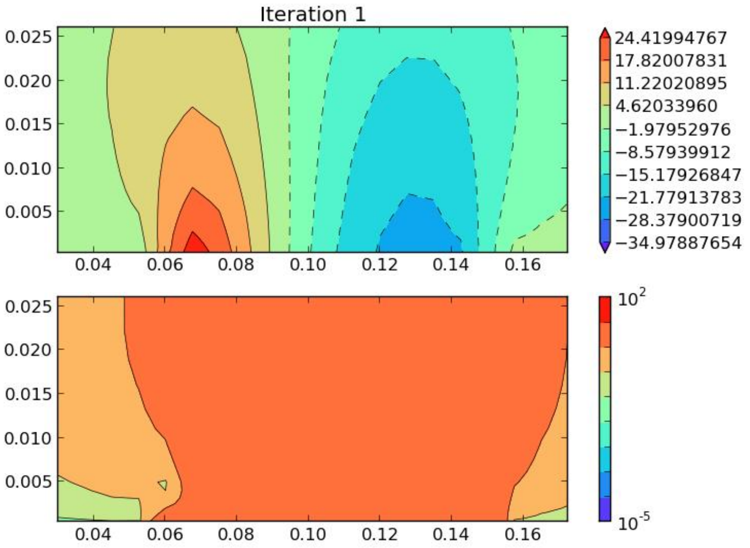
Evolution of the cost functional with respect to the number of iterations

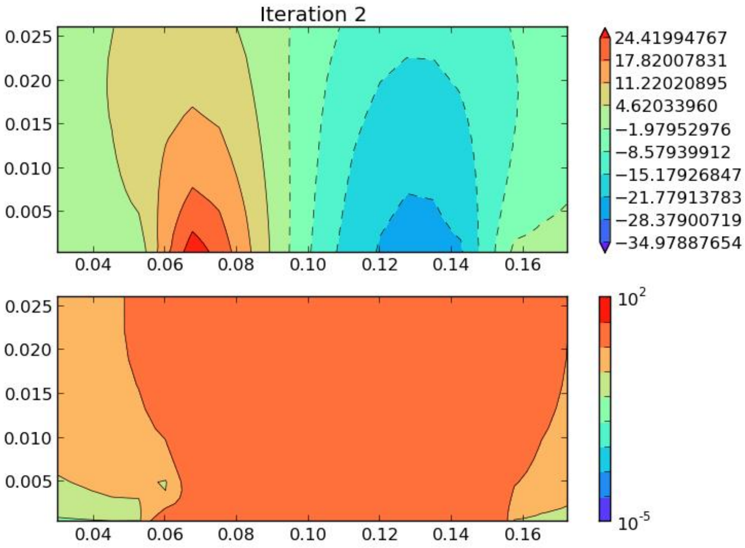


Evolution of the cost functional with respect to the number of iterations

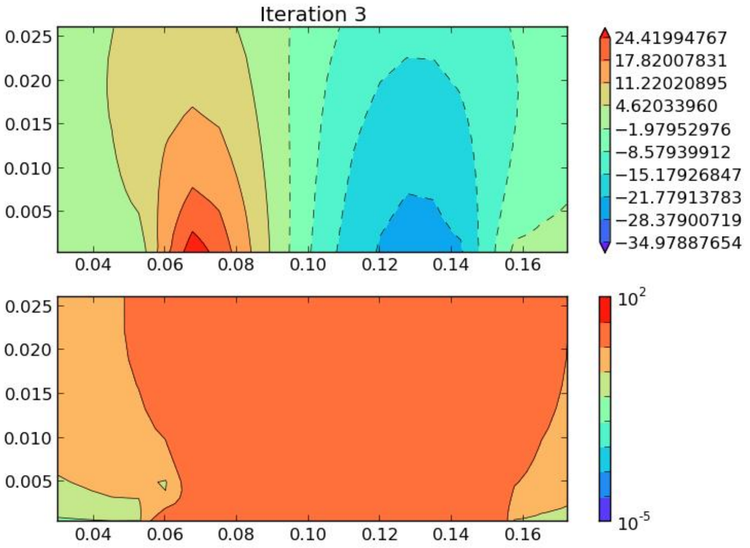


Convergence towards the measurements noise 0%

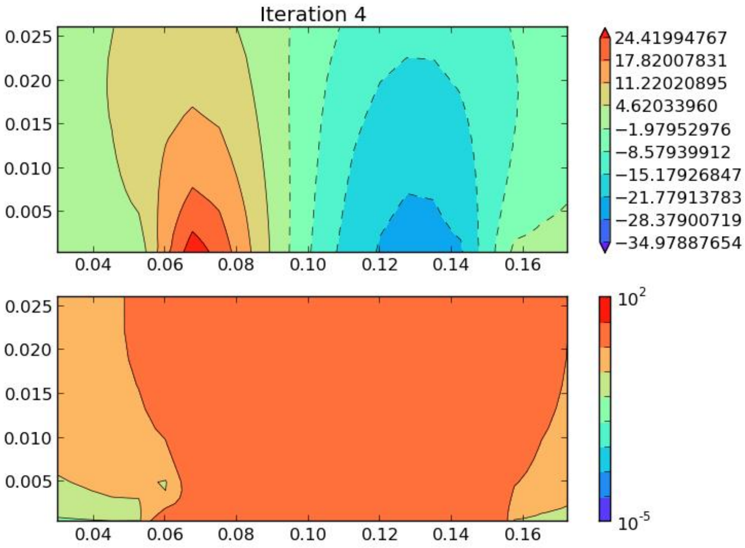



Iteration 2
0.025
0.020
0.015
0.010
0.005
0.04
0.06
0.08
0.10
0.12
0.14
0.16
24.41994767
17.82007831
11.22020895
4.62033960
−1.97952976
−8.57939912
−15.17926847
−21.77913783
−28.37900719
−34.97887654
10^2
10^{-5}

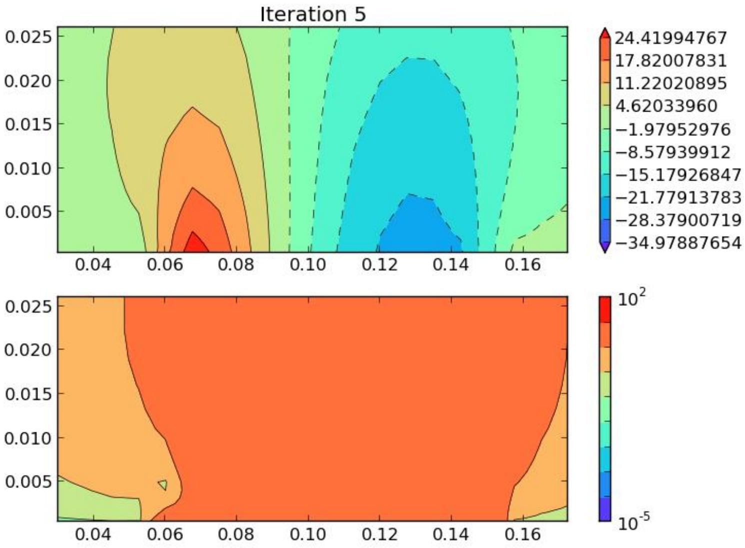
Convergence towards the measurements noise 0%



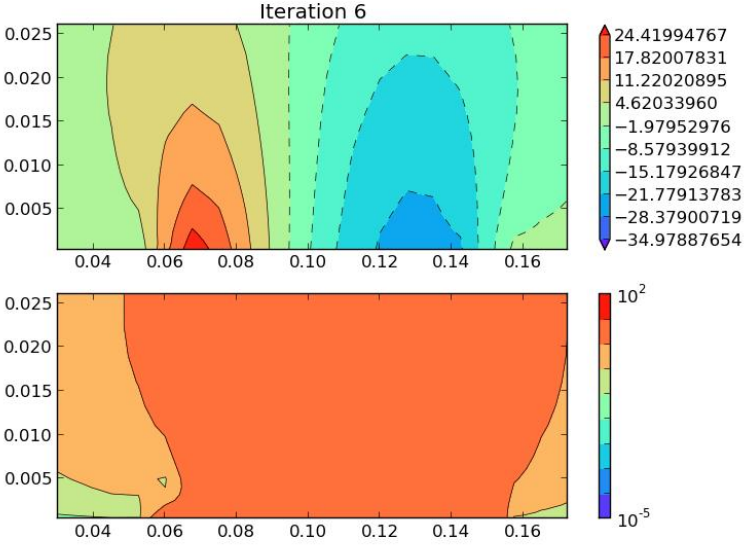
Convergence towards the measurements noise 0%



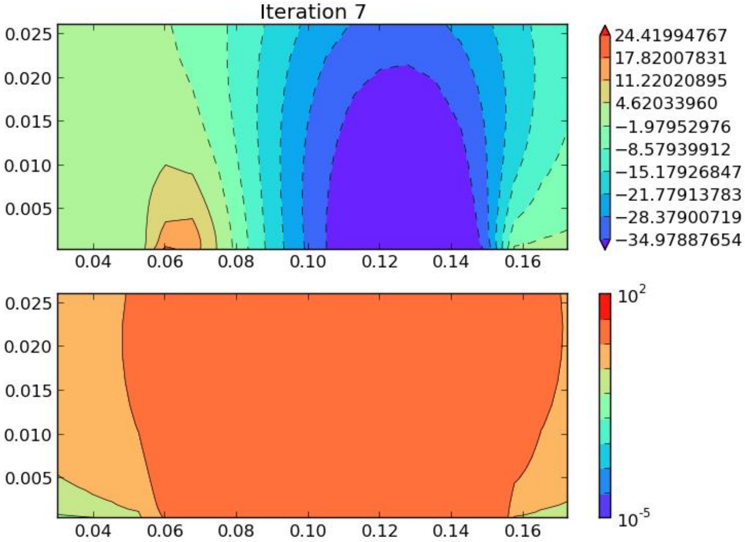
Convergence towards the measurements noise 0%



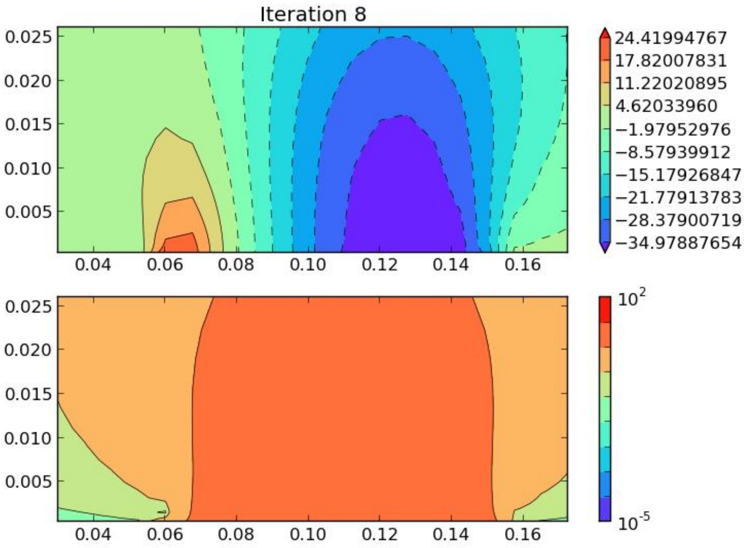
Convergence towards the measurements noise 0%



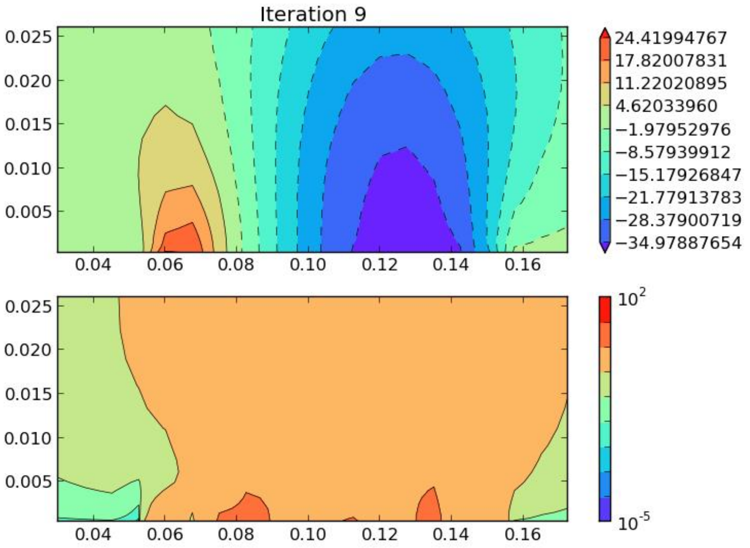
Convergence towards the measurements noise 0%



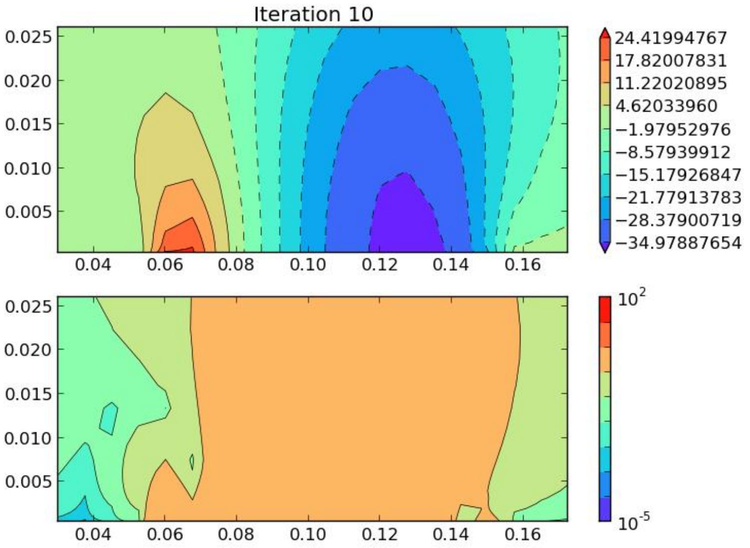
Convergence towards the measurements noise 0%



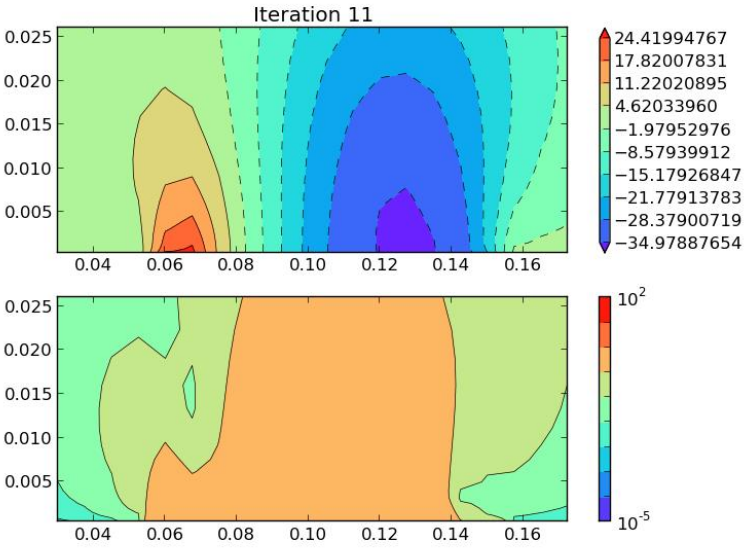
Convergence towards the measurements noise 0%



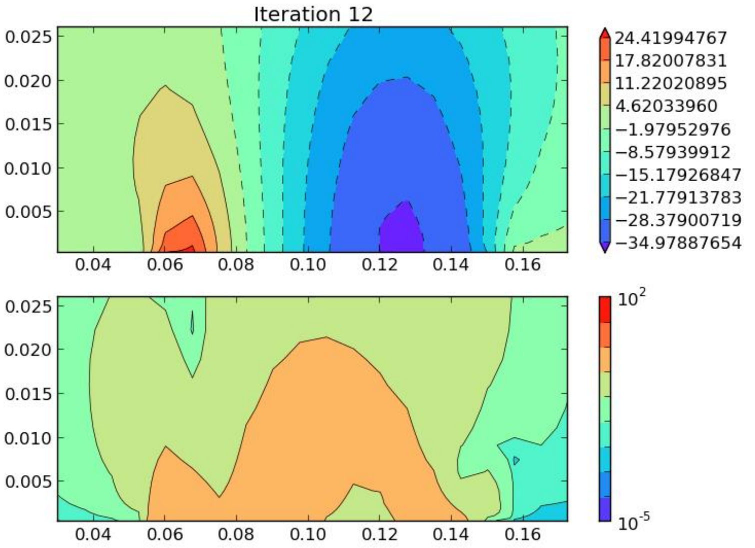
Convergence towards the measurements noise 0%



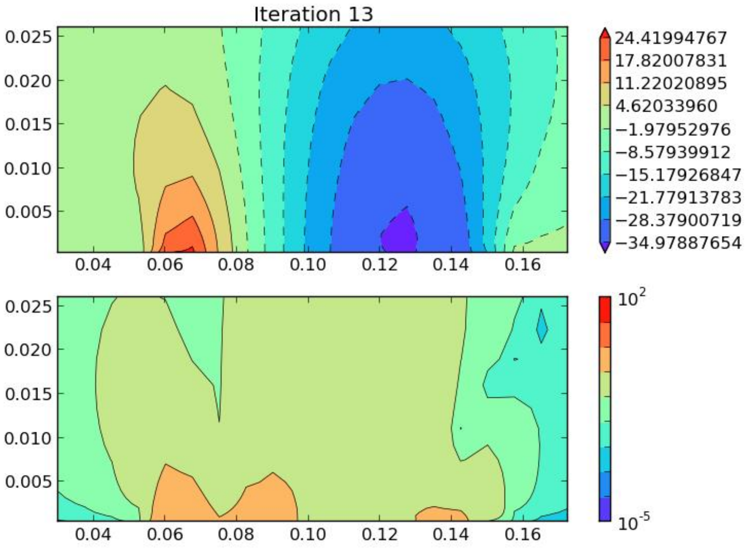
Convergence towards the measurements noise 0%



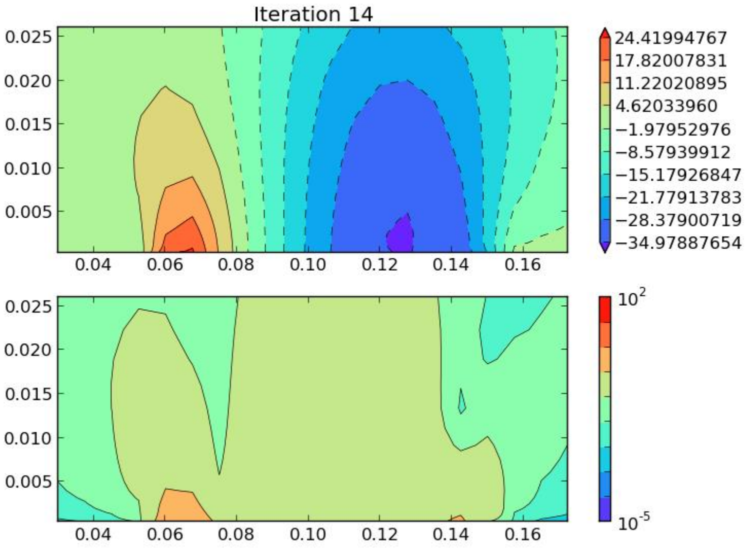
Convergence towards the measurements noise 0%



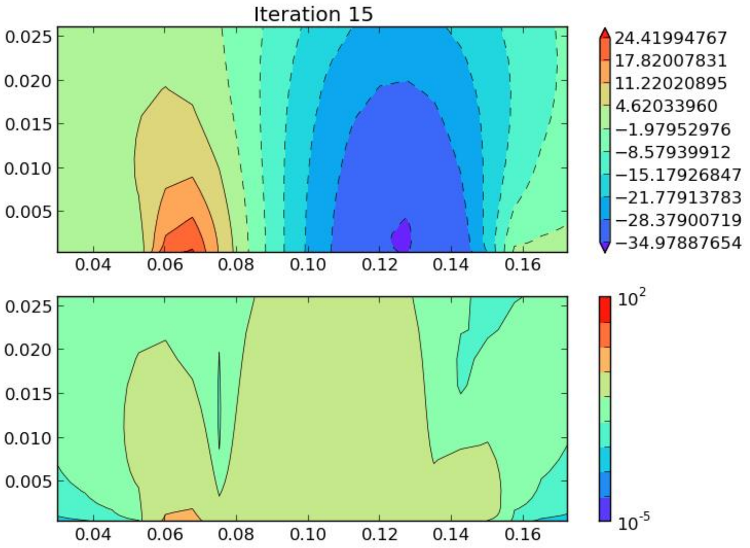
Convergence towards the measurements noise 0%



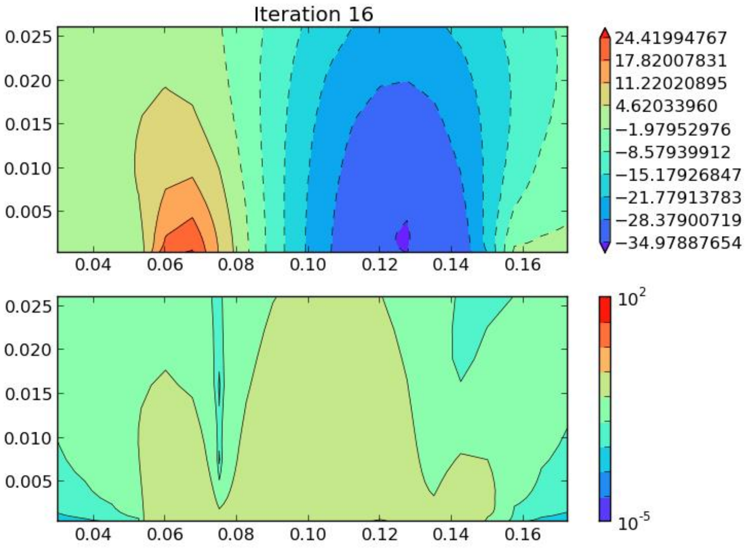
Convergence towards the measurements noise 0%



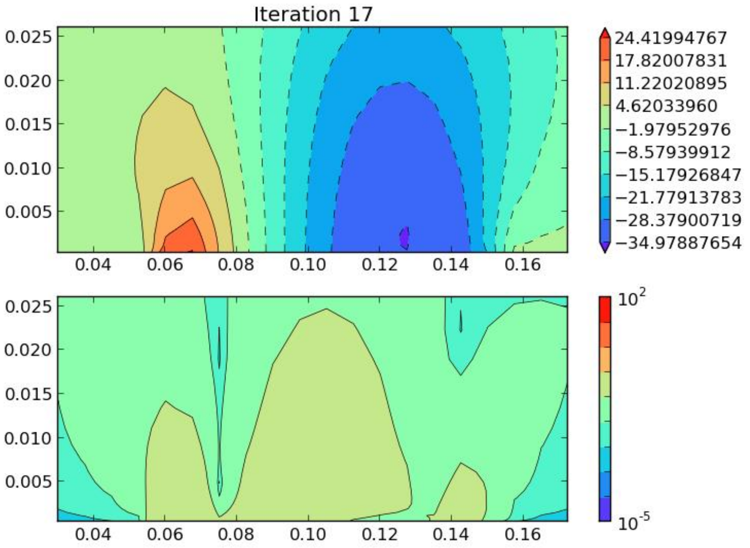
Convergence towards the measurements noise 0%



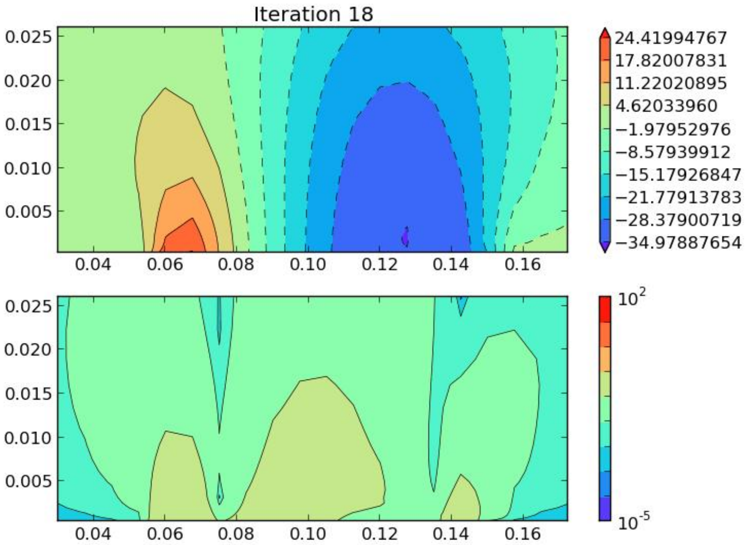
Convergence towards the measurements noise 0%



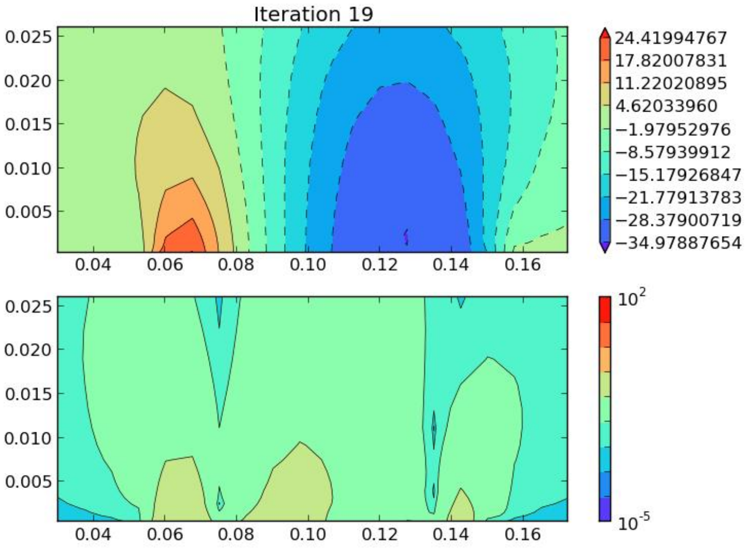
Convergence towards the measurements noise 0%



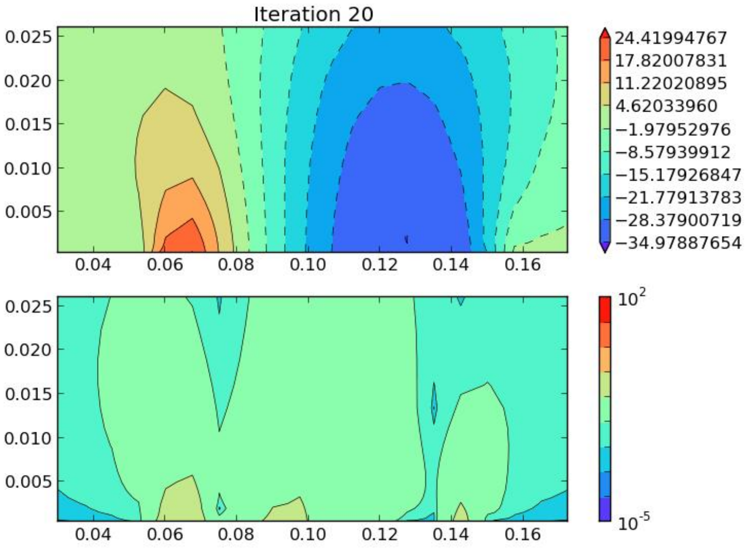
Convergence towards the measurements noise 0%



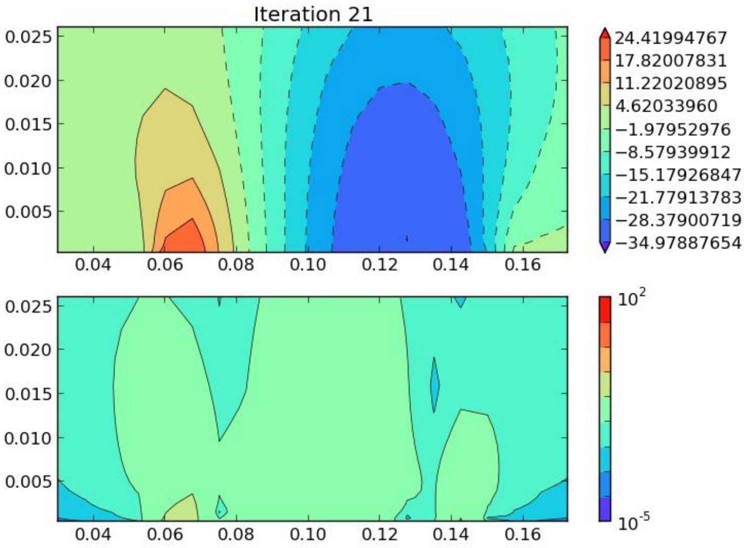
Convergence towards the measurements noise 0%



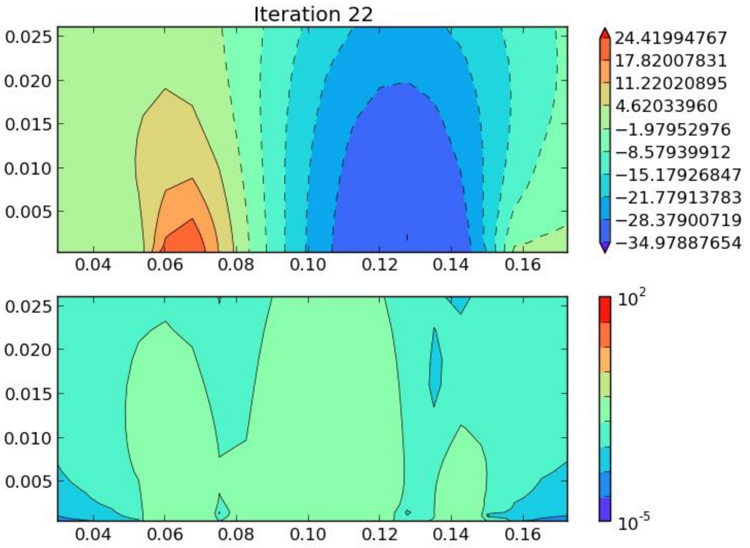
Convergence towards the measurements noise 0%



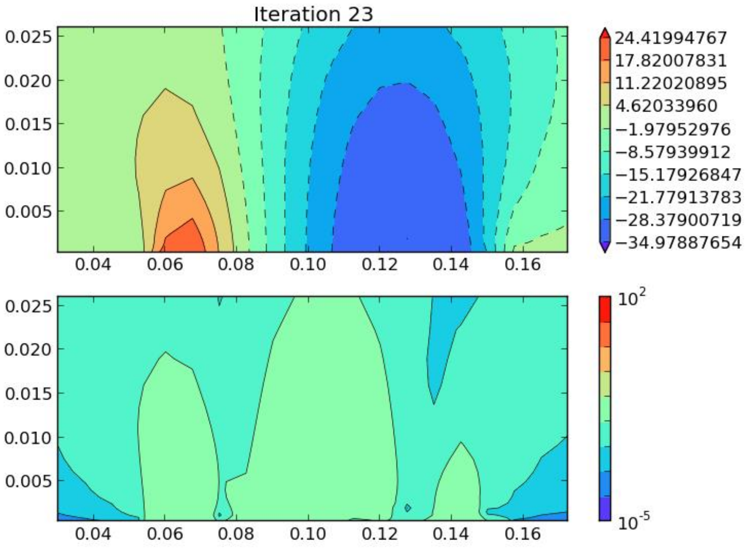
Convergence towards the measurements noise 0%



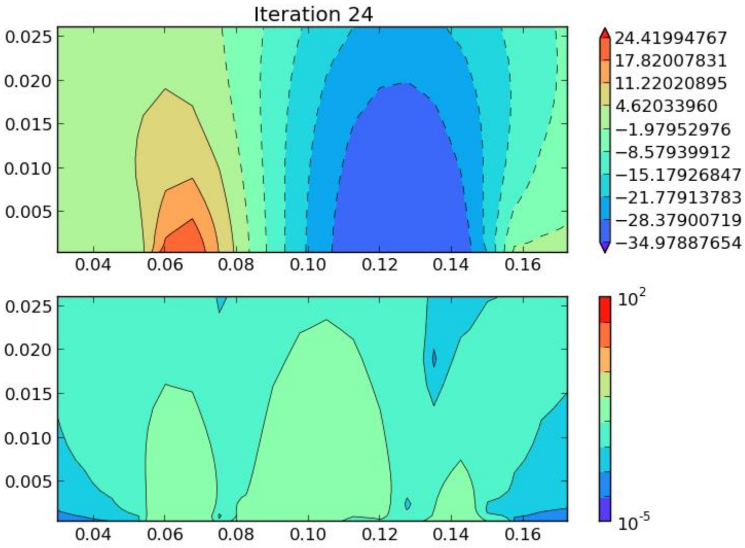
Convergence towards the measurements noise 0%



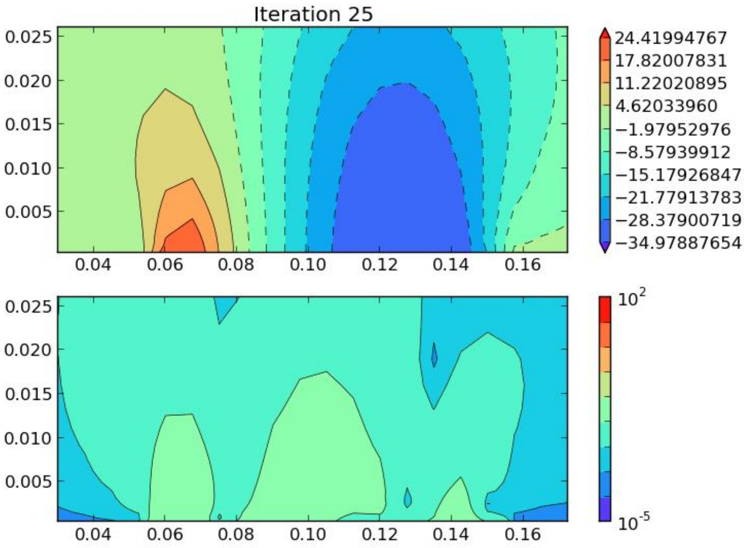
Convergence towards the measurements noise 0%



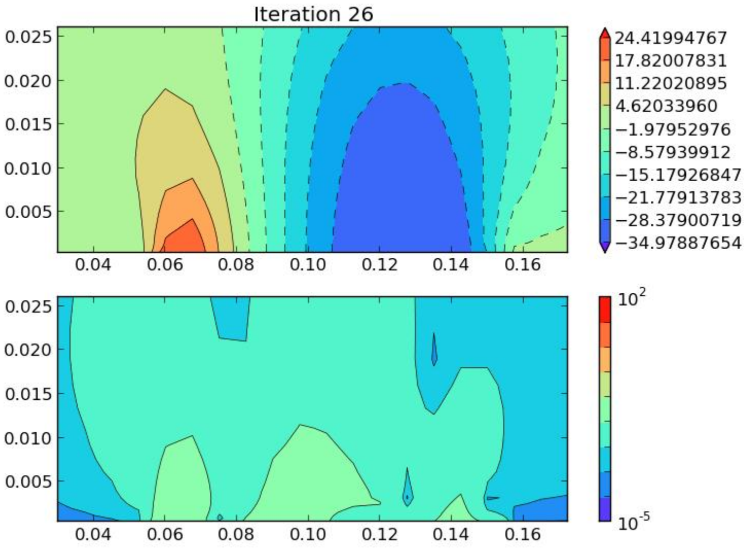
Convergence towards the measurements noise 0%



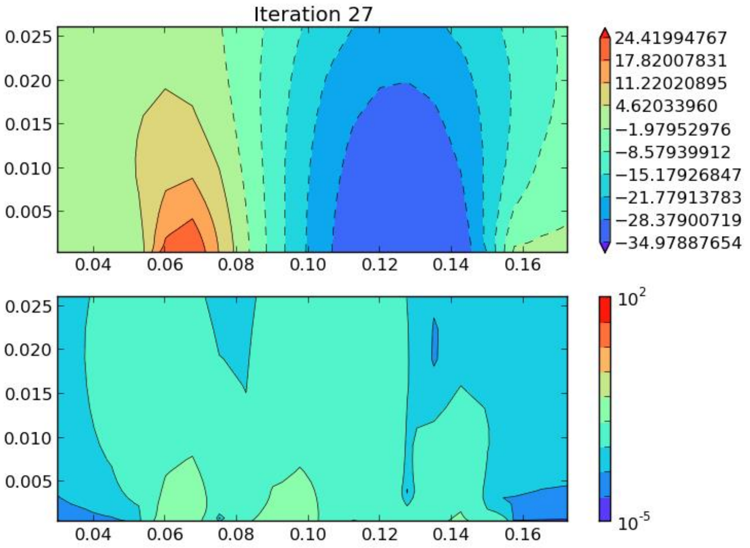
Convergence towards the measurements noise 0%



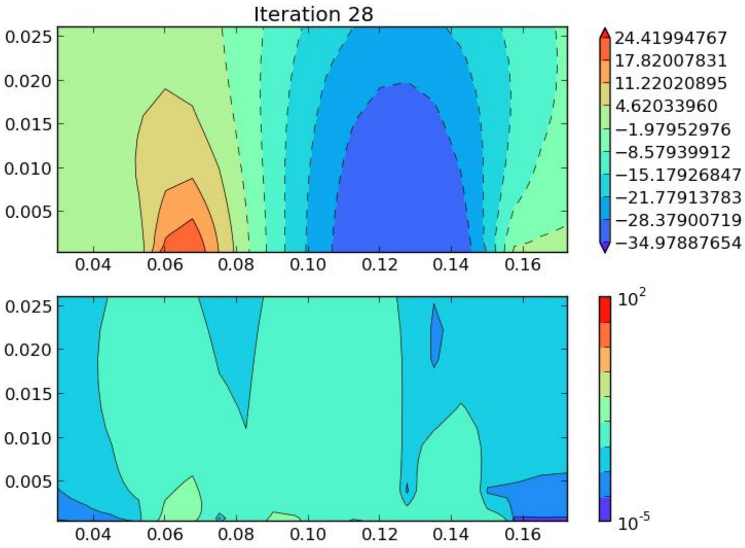
Convergence towards the measurements noise 0%



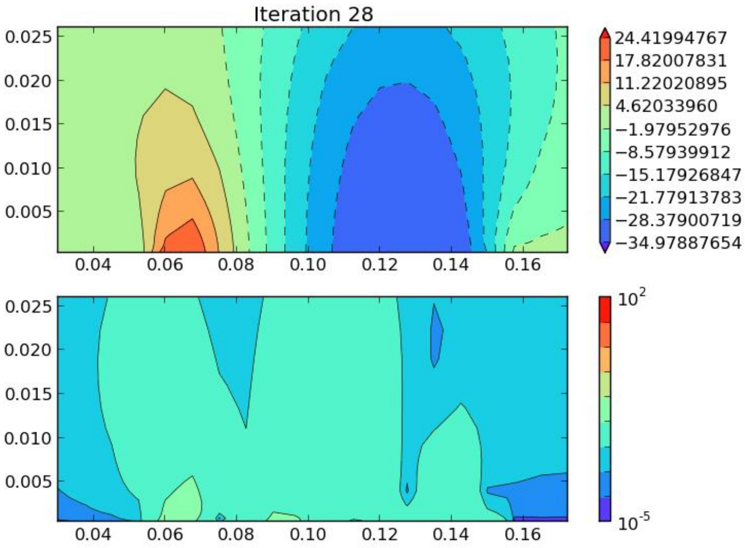
Convergence towards the measurements noise 0%



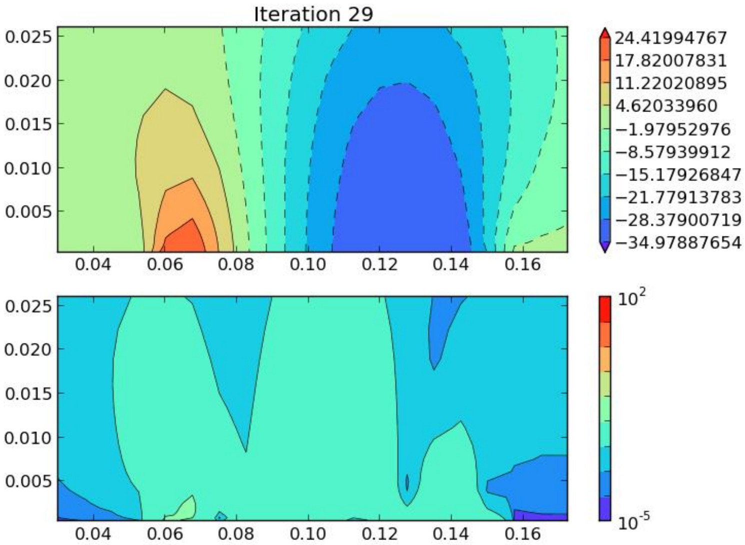
Convergence towards the measurements noise 0%



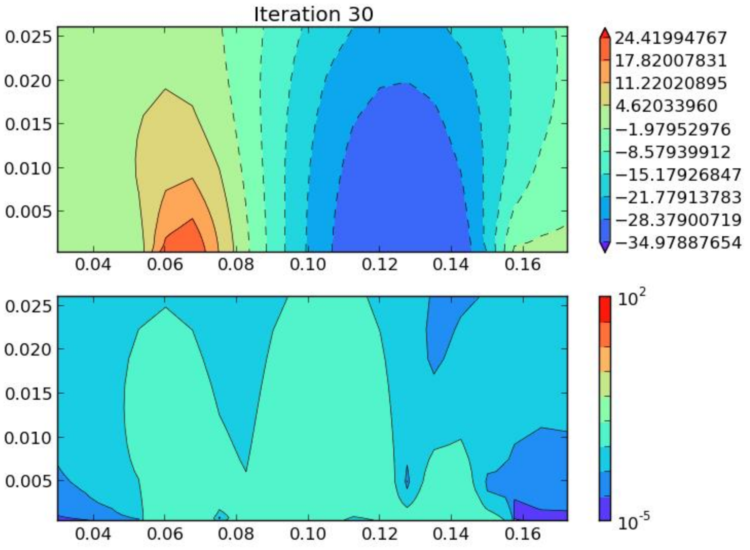
Convergence towards the measurements noise 0%



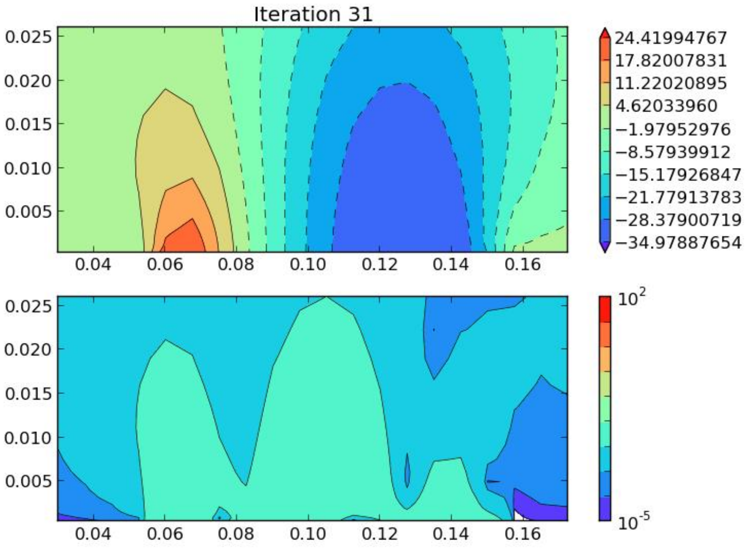
Convergence towards the measurements noise 0%



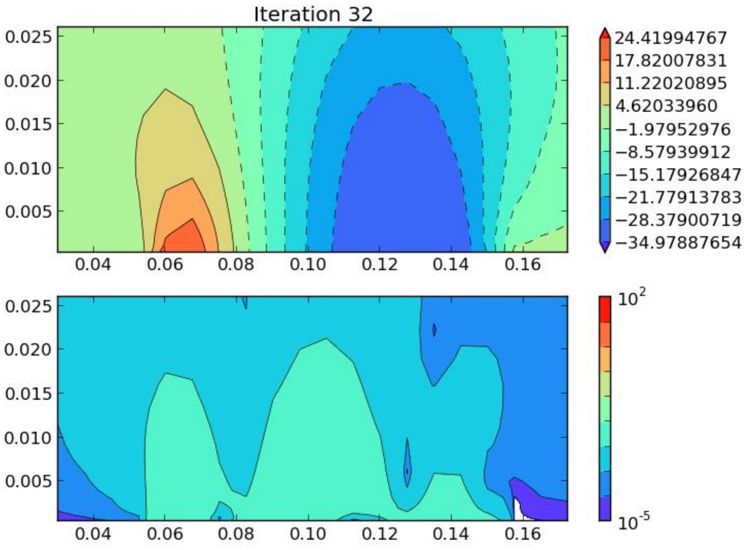
Convergence towards the measurements noise 0%



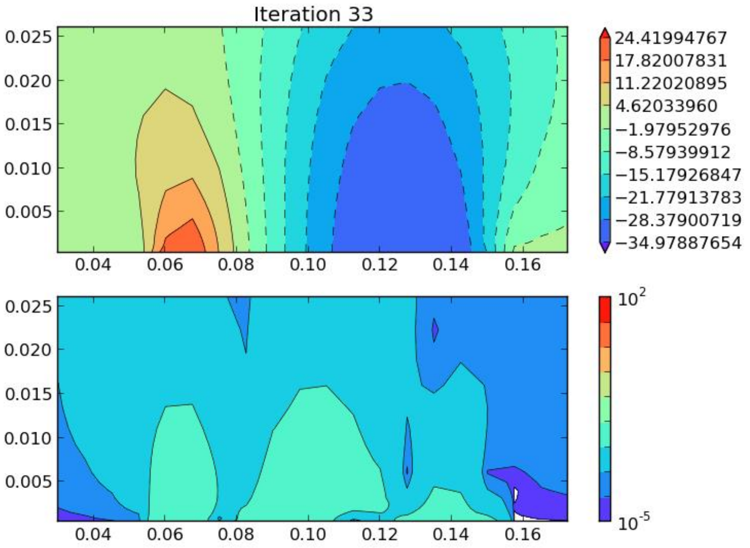
Convergence towards the measurements noise 0%



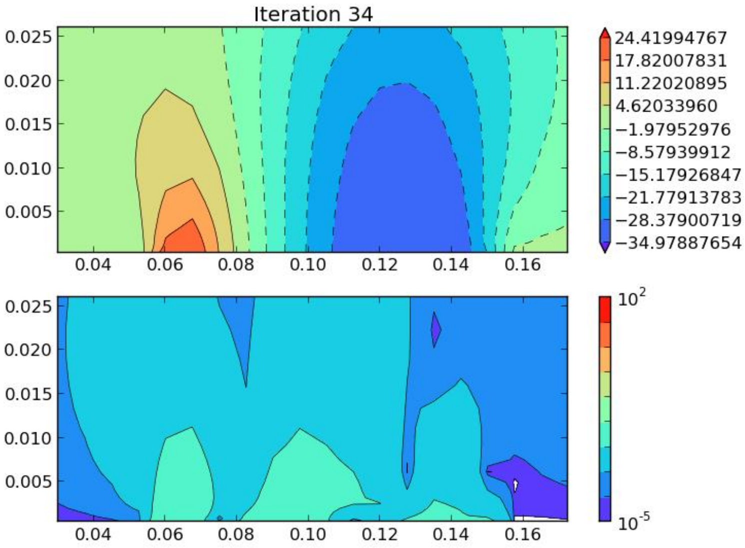
Convergence towards the measurements noise 0%



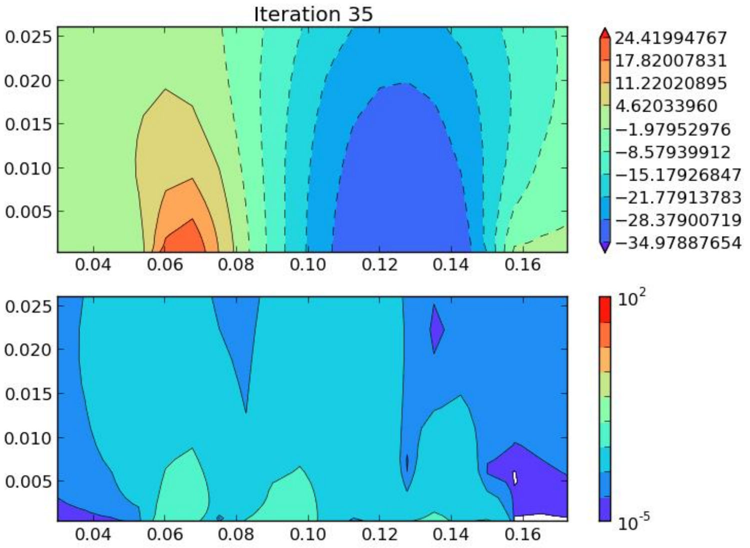
Convergence towards the measurements noise 0%



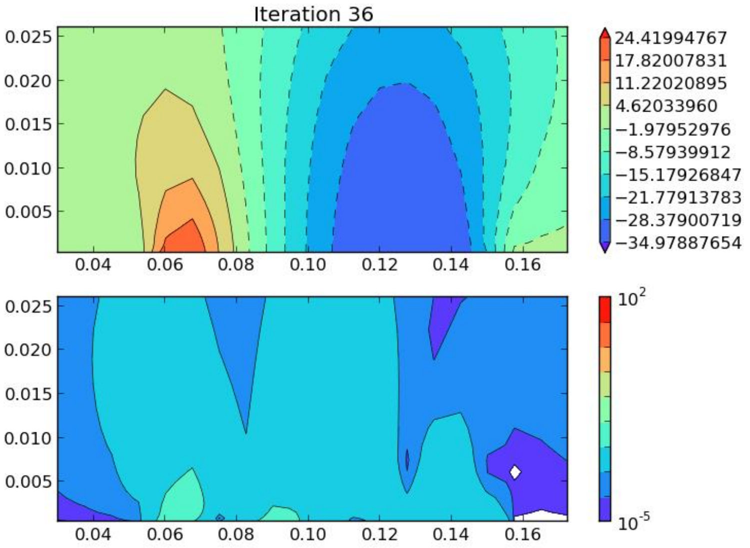
Convergence towards the measurements noise 0%



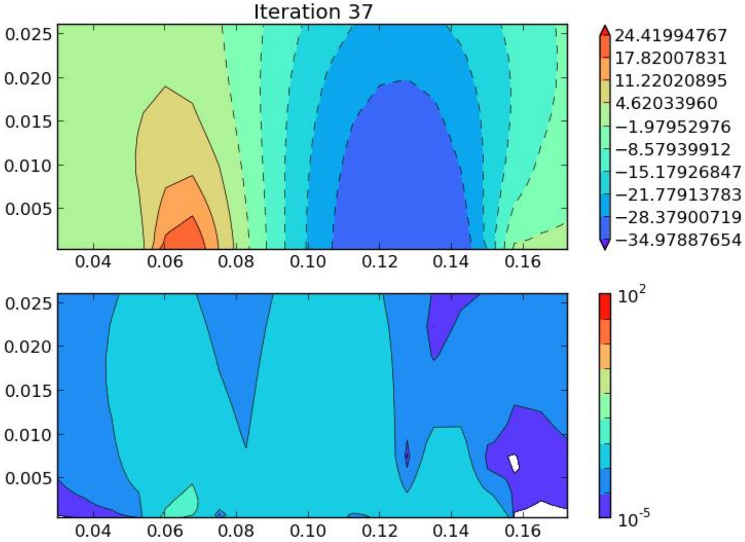
Convergence towards the measurements noise 0%



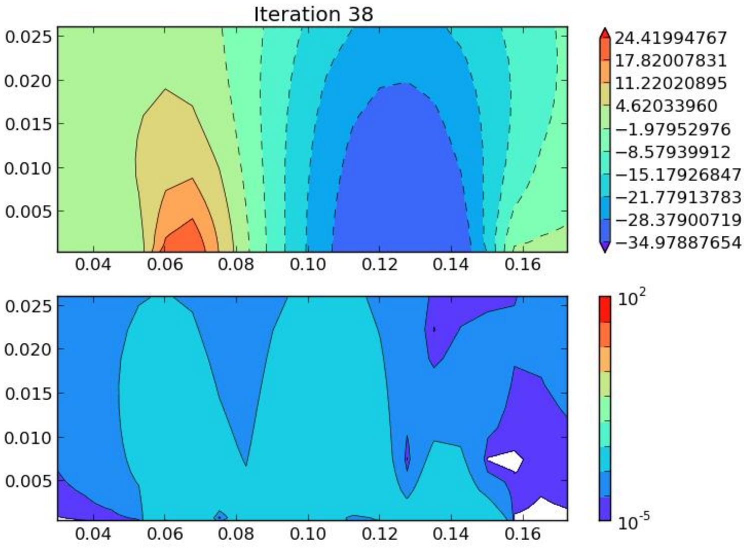
Convergence towards the measurements noise 0%



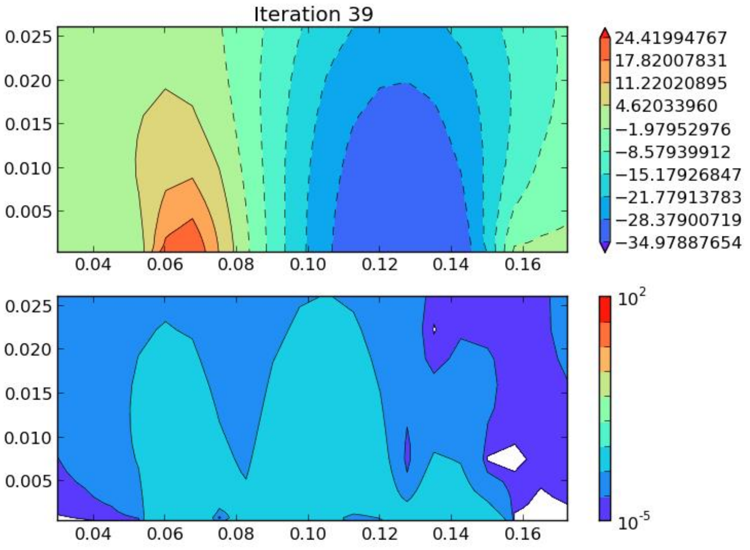
Convergence towards the measurements noise 0%



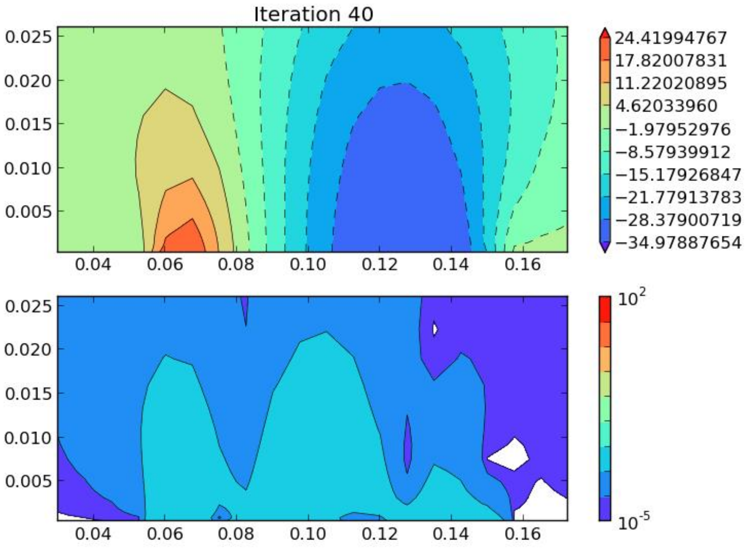
Convergence towards the measurements noise 0%



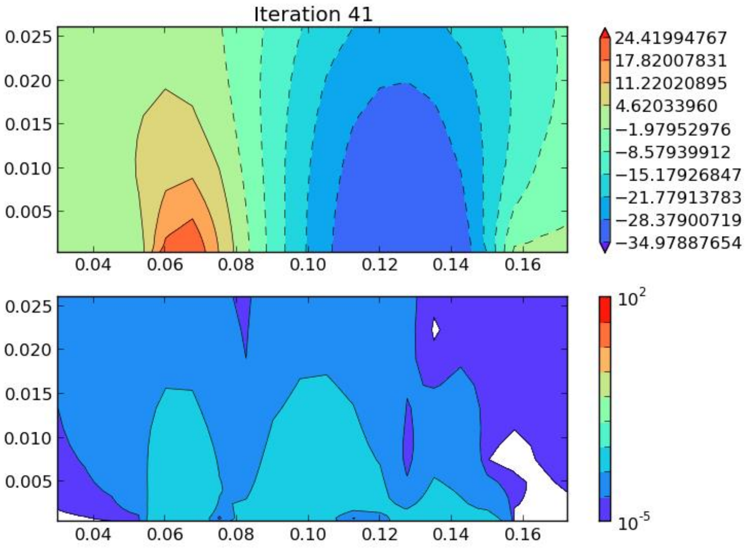
Convergence towards the measurements noise 0%



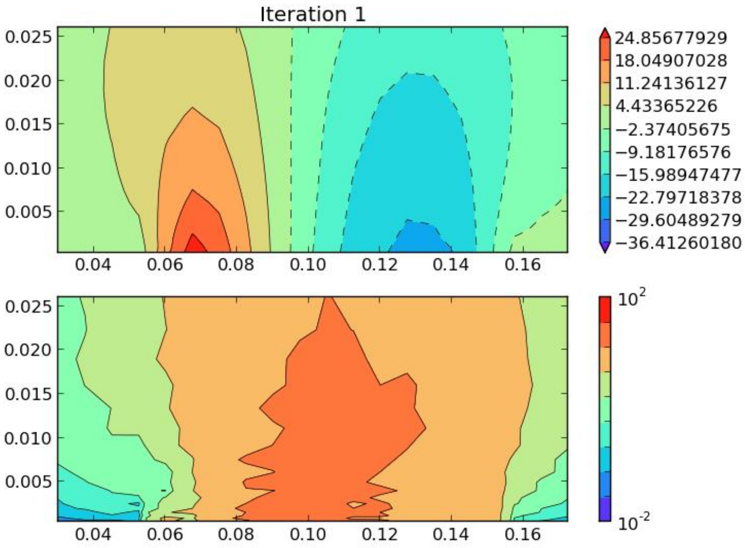
Convergence towards the measurements noise 0%



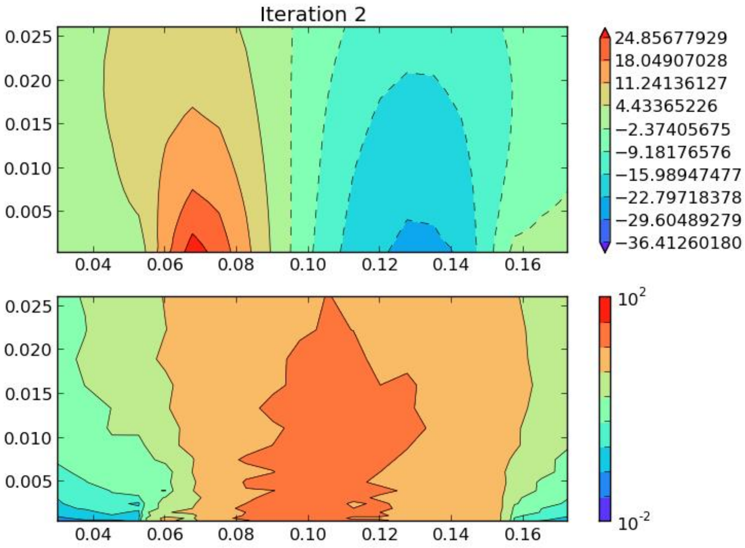
Convergence towards the measurements noise 0%



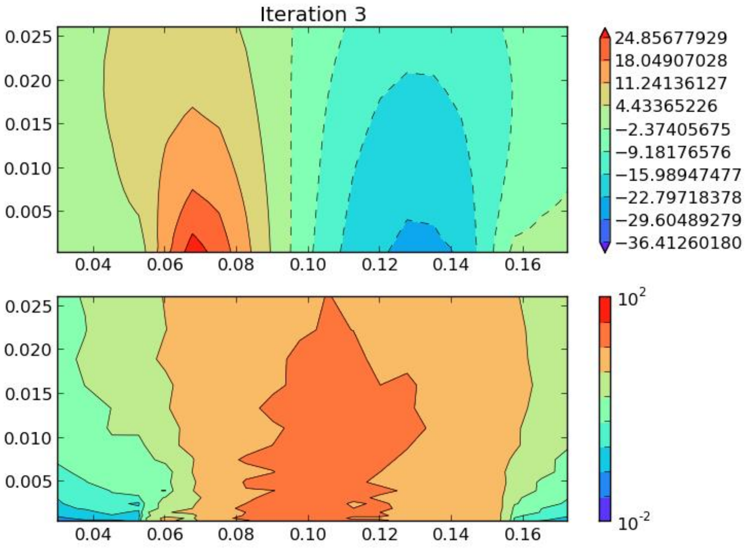
Convergence toward the measures, noise = 10%



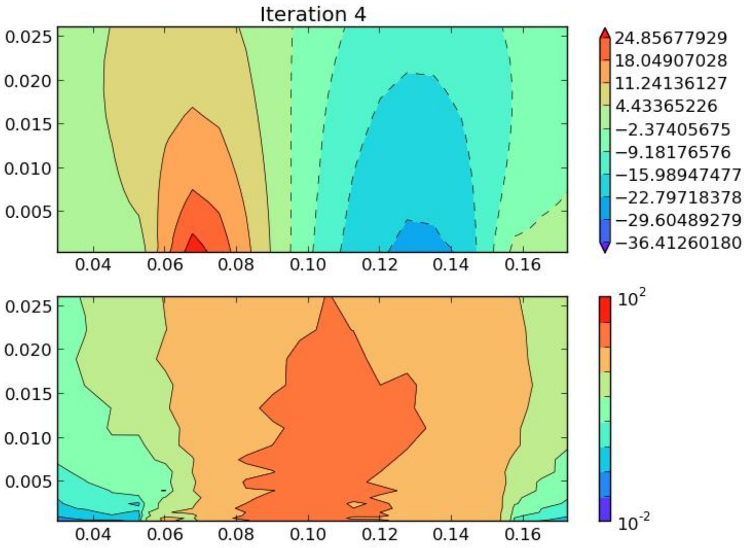
Convergence toward the measures, noise $= 10\%$



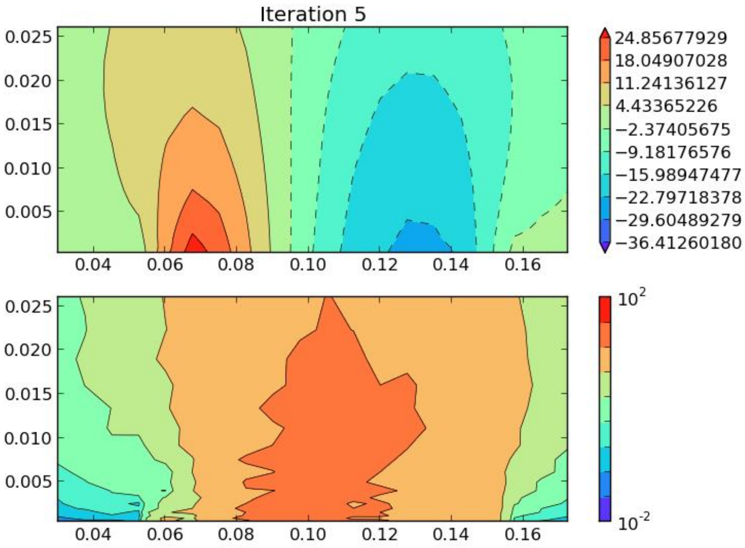
Convergence toward the measures, noise = 10%



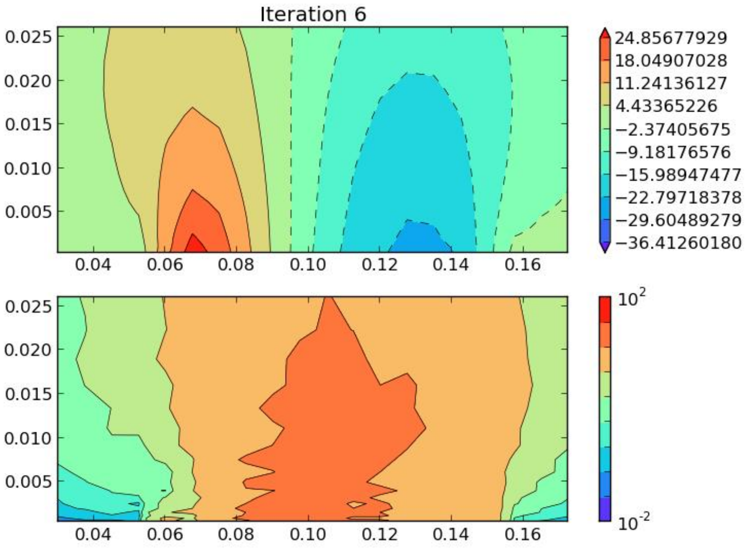
Convergence toward the measures, noise = 10%



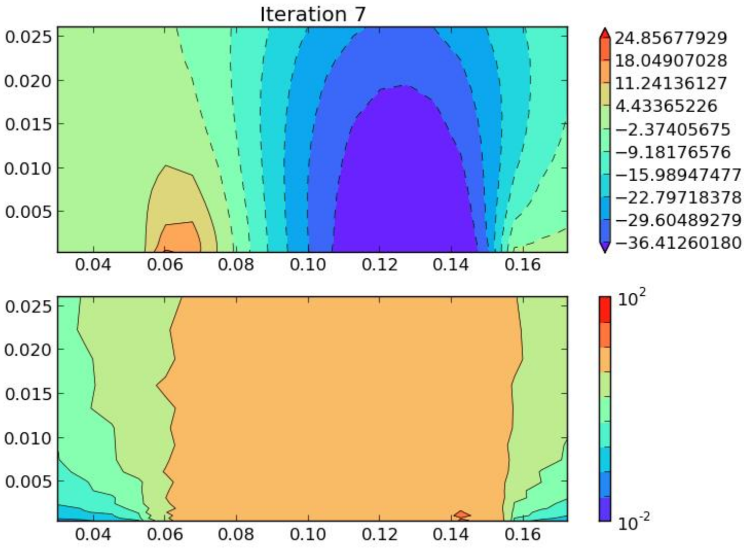
Convergence toward the measures, noise = 10%



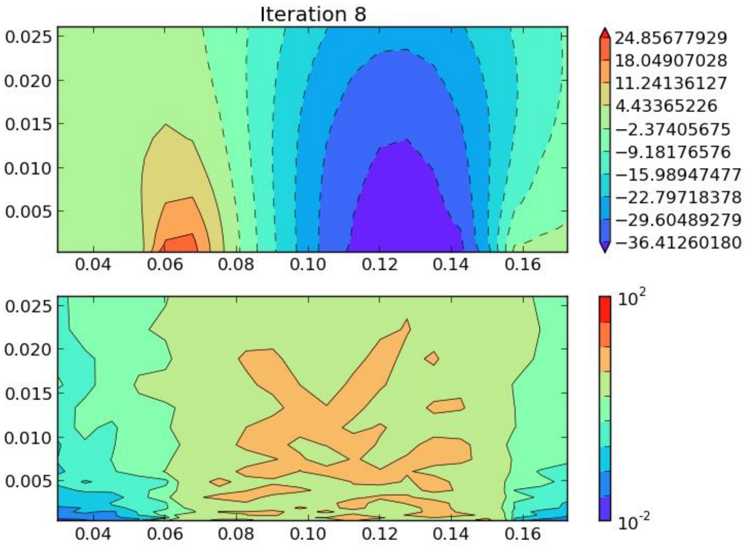
Convergence toward the measures, noise = 10%



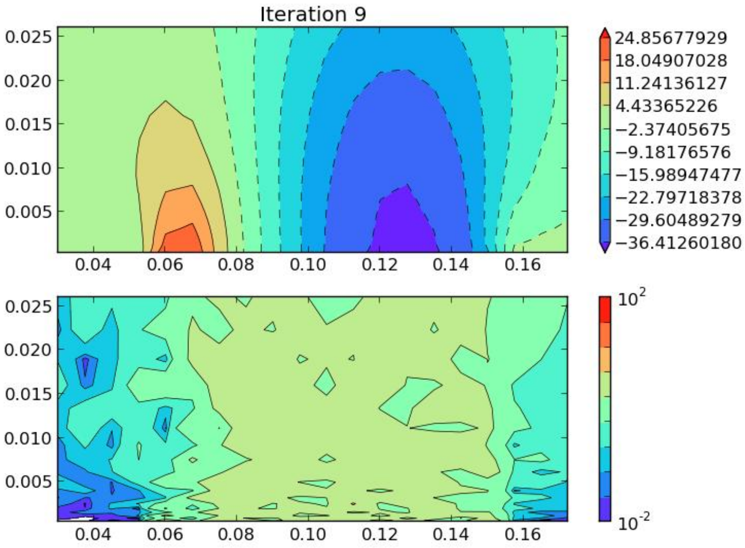
Convergence toward the measures, noise = 10%



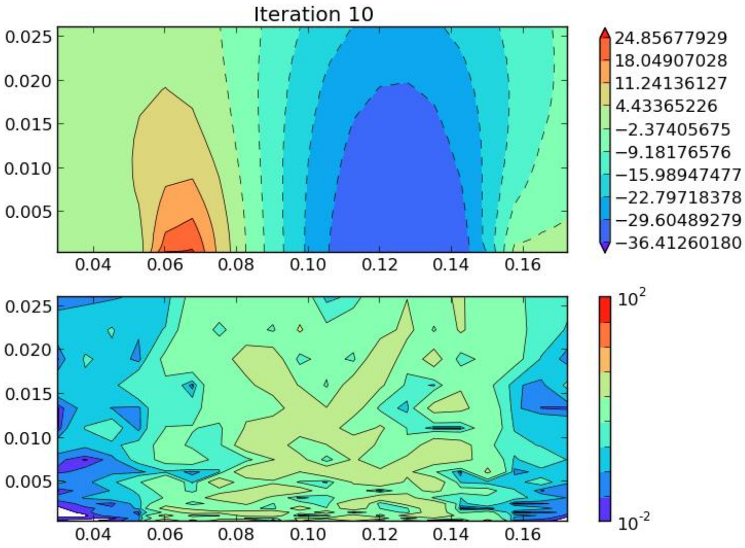
Convergence toward the measures, noise = 10%



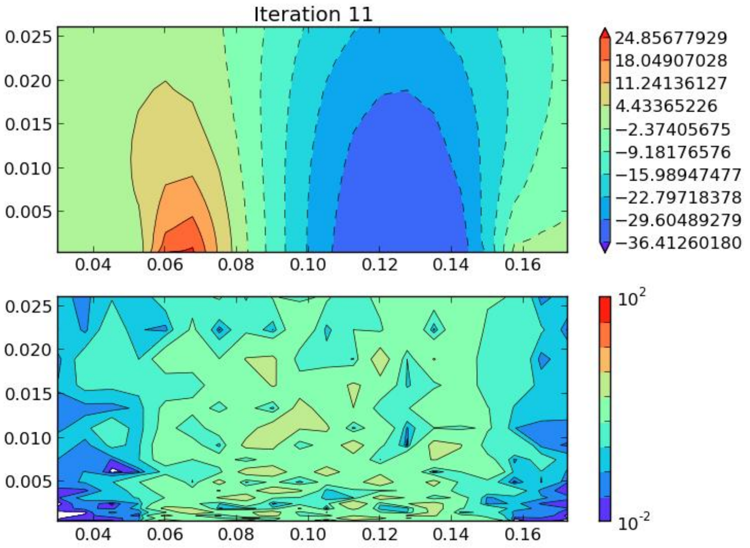
Convergence toward the measures, noise = 10%



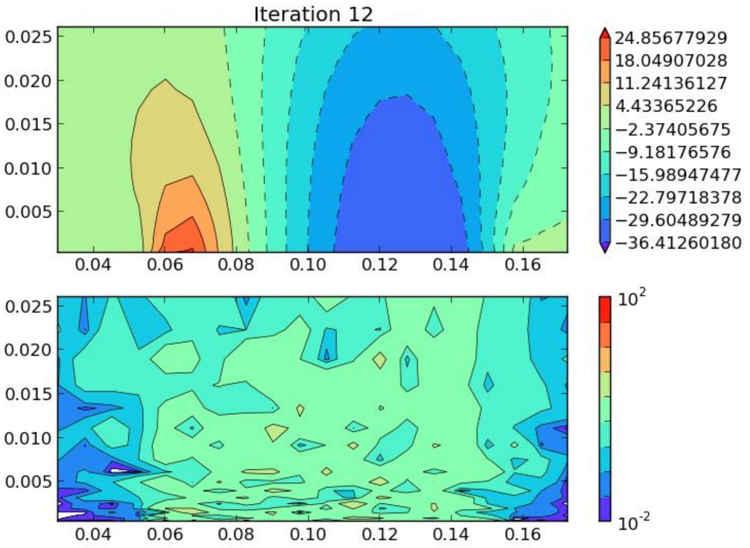
Convergence toward the measures, noise = 10%



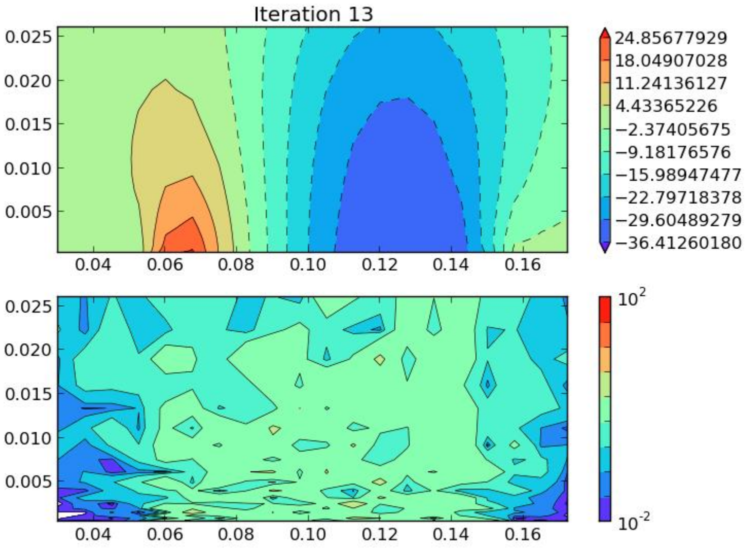
Convergence toward the measures, noise = 10%



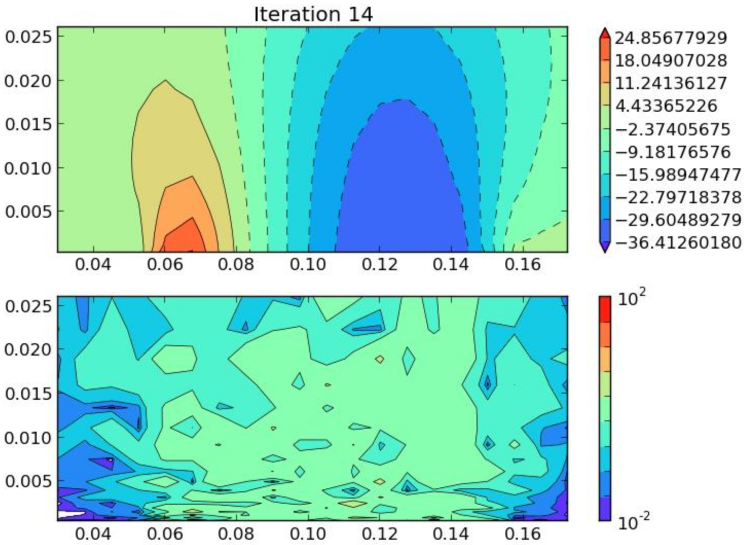
Convergence toward the measures, noise = 10%

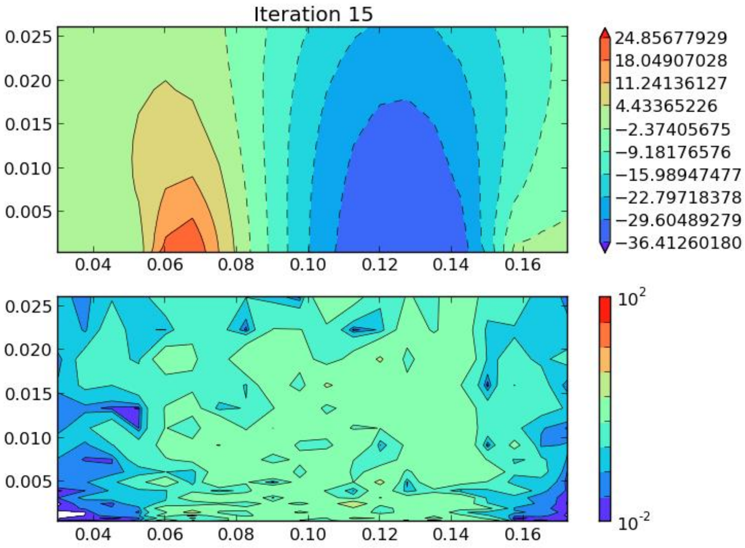


Convergence toward the measures, noise = 10%

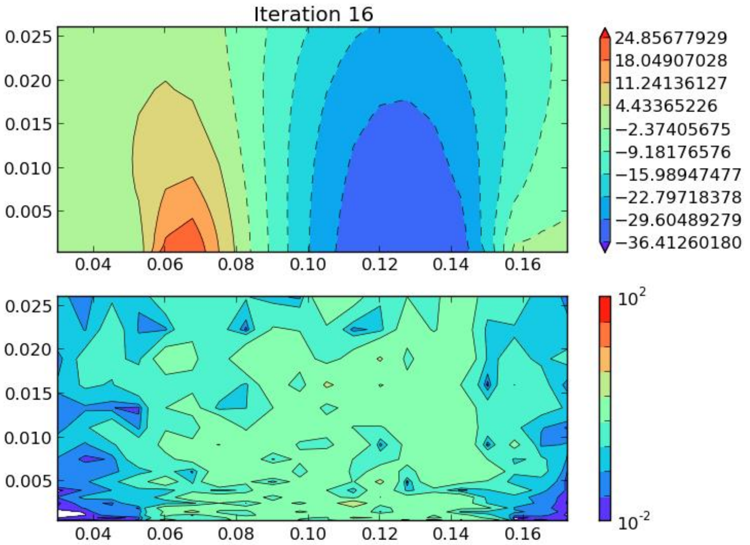


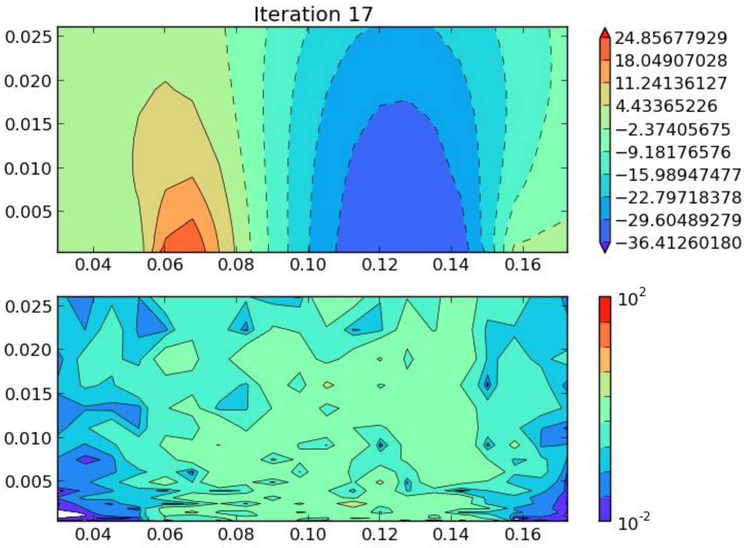
Convergence toward the measures, noise = 10%



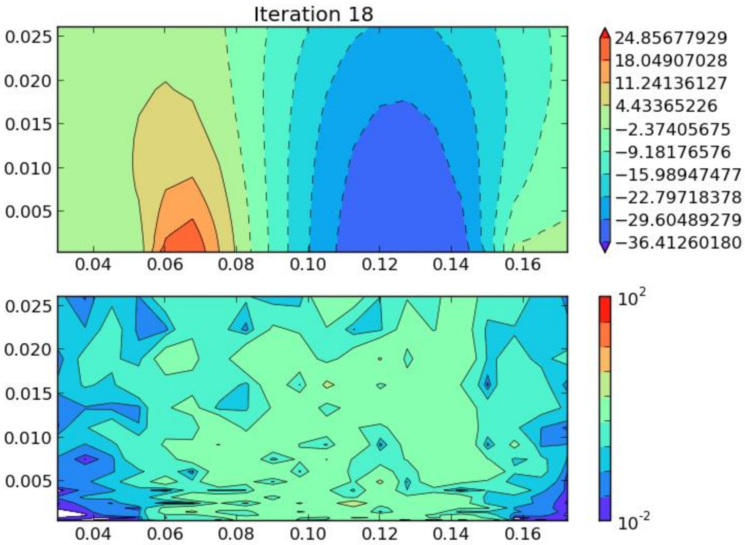

Iteration 15
0.025
0.020
0.015
0.010
0.005
0.04
0.06
0.08
0.10
0.12
0.14
0.16
24.85677929
18.04907028
11.24136127
4.43365226
−2.37405675
−9.18176576
−15.98947477
−22.79718378
−29.60489279
−36.41260180
10^{2}
10^{-2}

Convergence toward the measures, noise = 10%

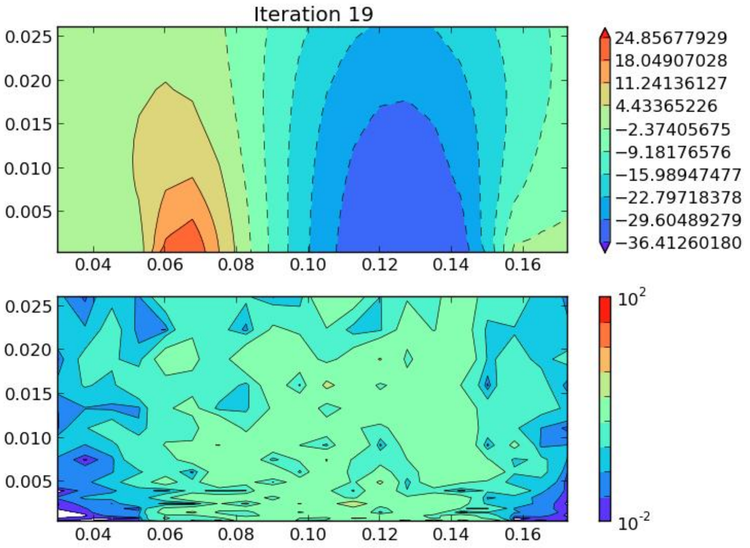



Iteration 17
0.025
0.020
0.015
0.010
0.005
0.04
0.06
0.08
0.10
0.12
0.14
0.16
24.85677929
18.04907028
11.24136127
4.43365226
−2.37405675
−9.18176576
−15.98947477
−22.79718378
−29.60489279
−36.41260180
0.025
0.020
0.015
0.010
0.005
0.04
0.06
0.08
0.10
0.12
0.14
0.16
10^2
10^-2

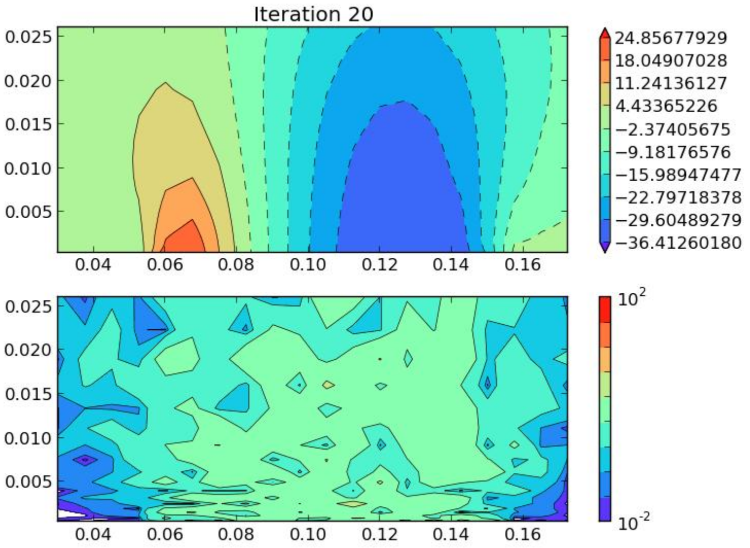
Convergence toward the measures, noise = 10%



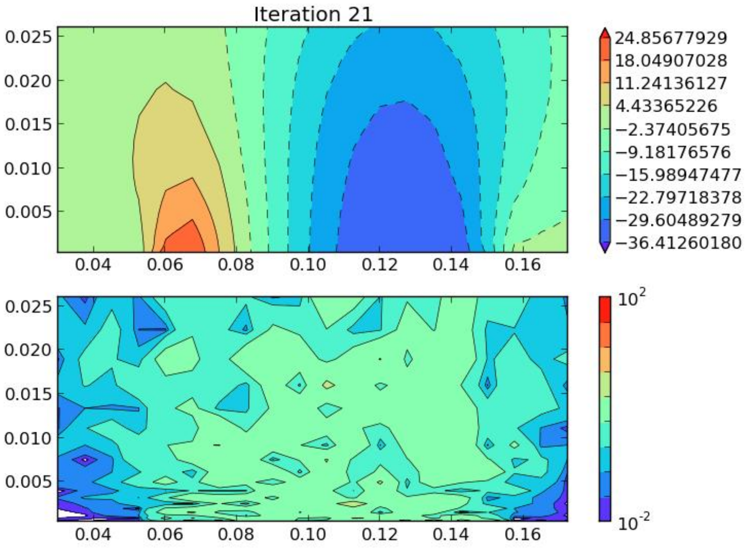
Convergence toward the measures, noise = 10%



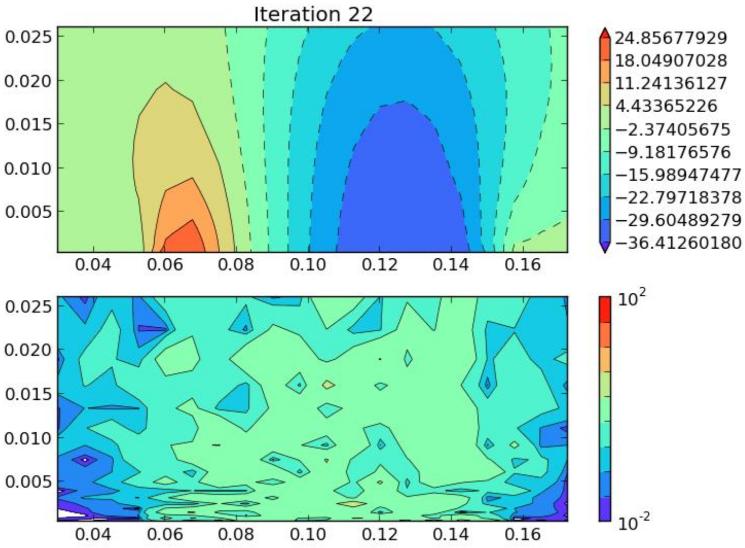
Convergence toward the measures, noise = 10%



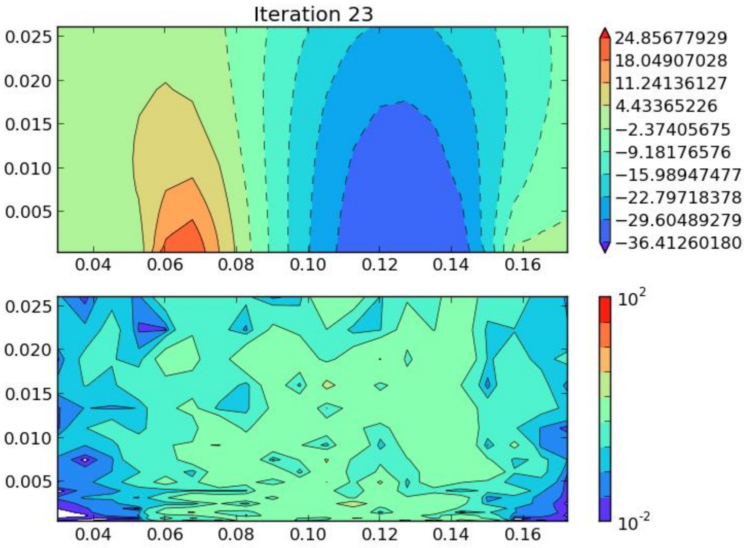
Convergence toward the measures, noise = 10%



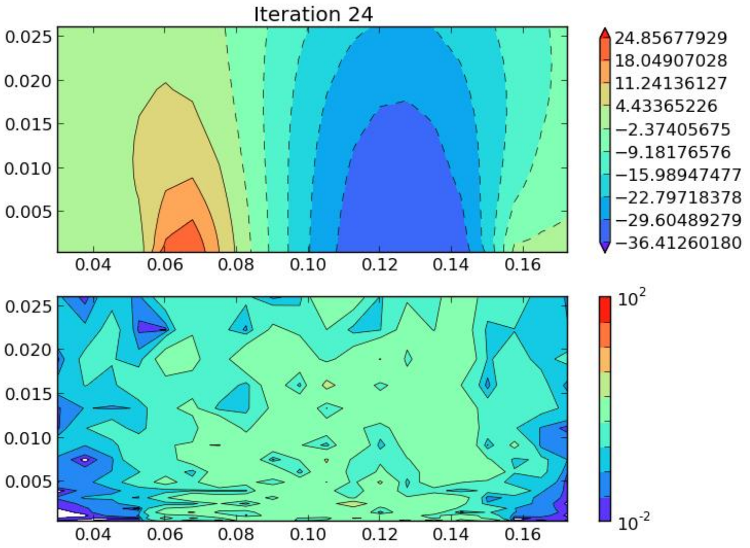
Convergence toward the measures, noise = 10%



Convergence toward the measures, noise = 10%



Convergence toward the measures, noise = 10%



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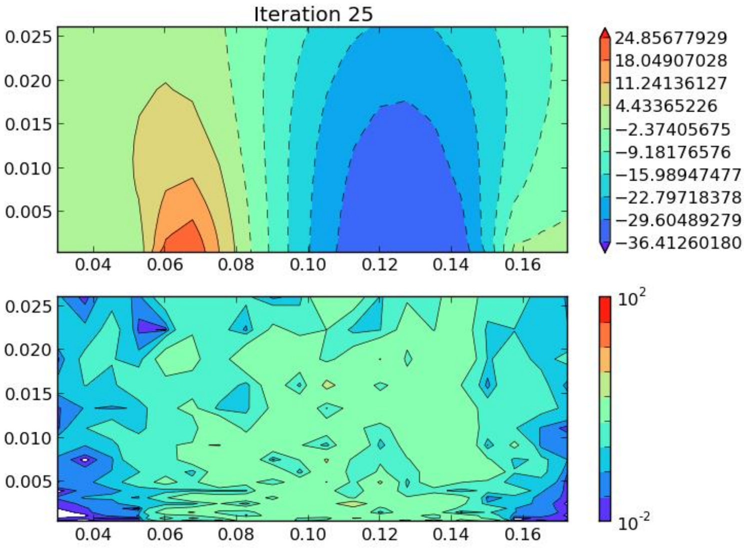

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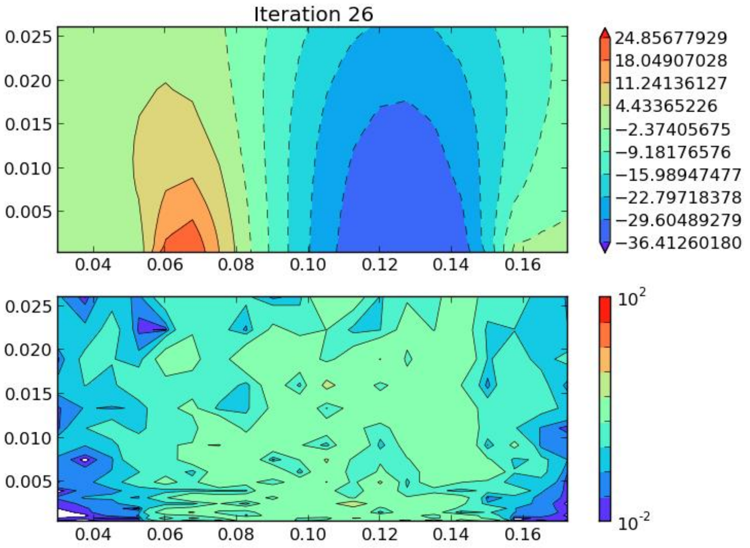

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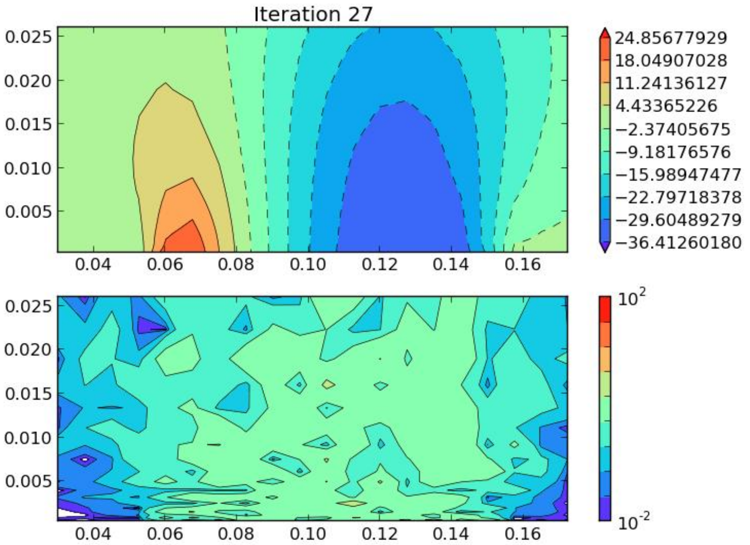

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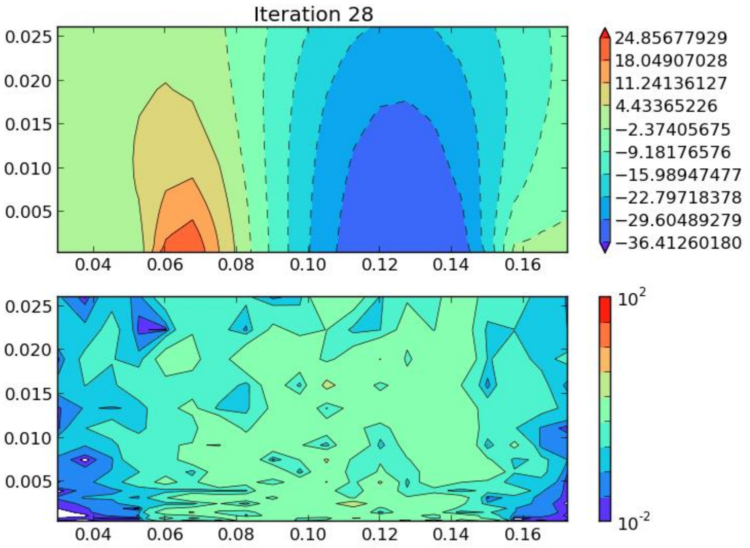

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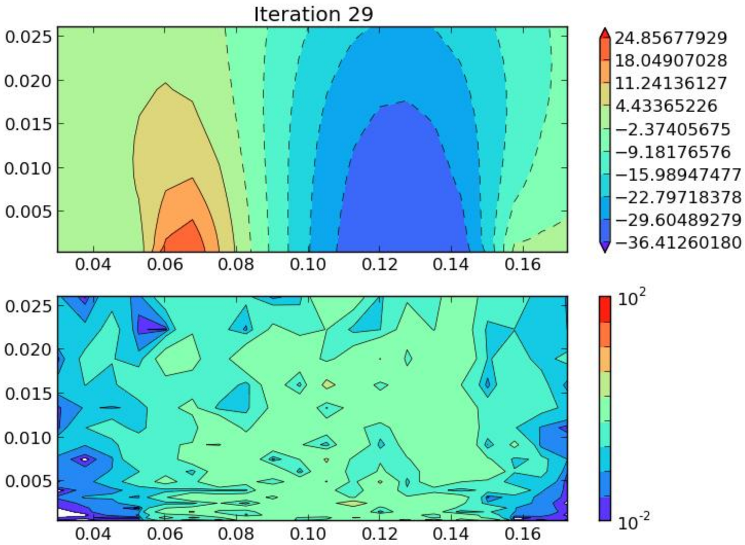

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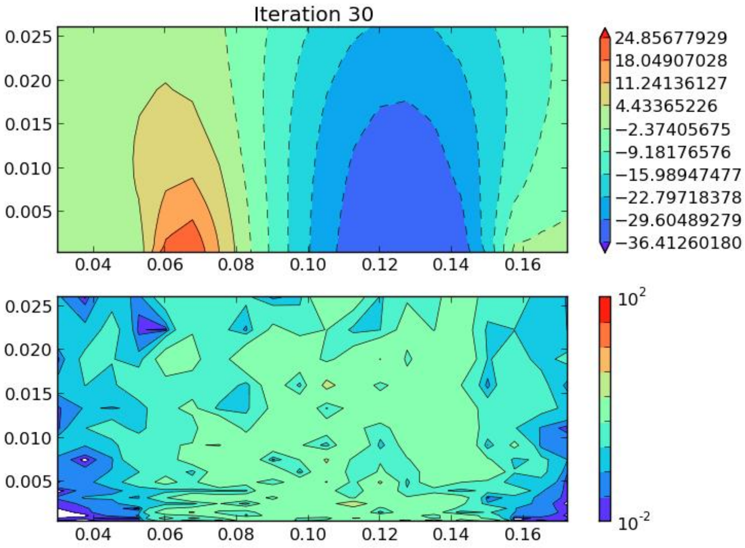

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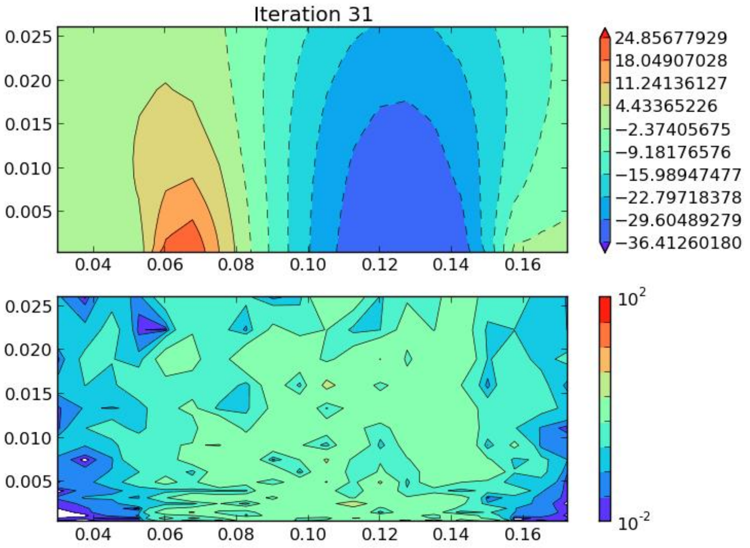

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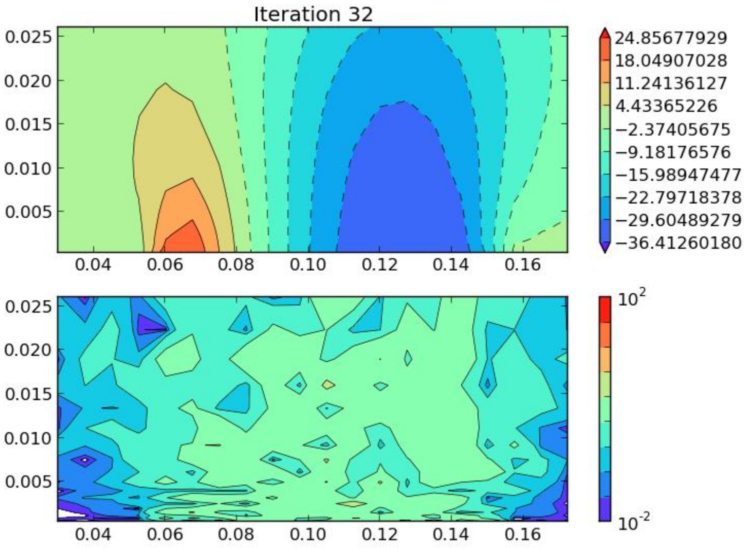

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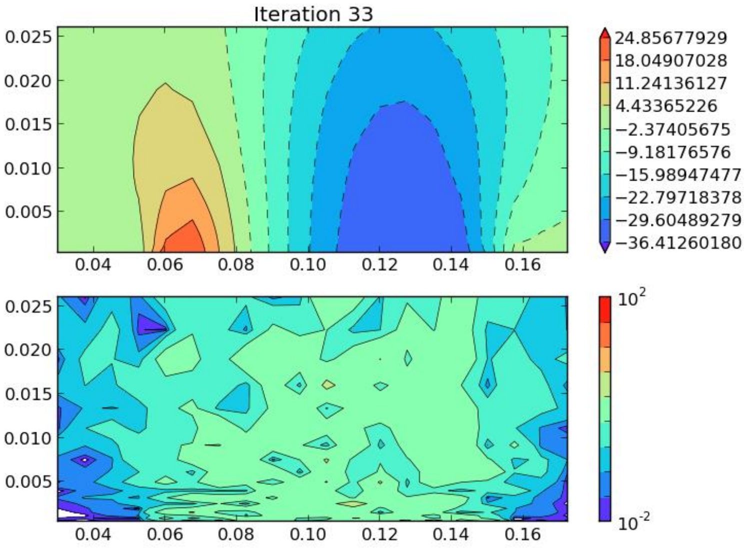

Thank you !