Serie 11

No. of Theoretical Exercises: 5.

No. of Practical Exercise: 0.

1. (Boundary Approximation)

Consider a curvilinear triangle K and a simple triangle K_1 with diameter h_{K_1} such that $K \subset K_1$ (or $K_1 \subset K$) with the same vertices (See Section 4.9.3 fig 101). Define $C = K - K_1$ (or $C = K_1 - K$) and let the maximum distance in boundaries of K and K_1 is $Ch^2_{K_1}$. Prove the following estimate:

$$\inf_{v_n \in V_n} |u - \tilde{v}_n|_{H^1(K)} \leq C h_{K_1} ||u||_{H^2(K)}$$

here $u \in H^2(K) \cap H^1_0(K)$ and $V_n(K_1)$ is the space of p.w. linear function on K_1 and $\tilde{v}_n$is extension of v_n to K.

Hint: For the case when $K_1 \subset K$ use H^2-extention theorem for u

2. (Linear Interpolation of root function in 1D)

Consider the root function $u = x^\lambda$ on the domain $\Omega =]0, 1[$. Define the equidistance mesh $\mathcal{M} = \{ih : i = 0, \ldots, n\}$ with meshwidth $h = \frac{1}{n}$. Let $\mathsf{I}_1 : C^0([0, 1]) \mapsto \mathcal{S}^0_1(\mathcal{M})$ piecewise linear interpolation operator. Prove the following estimate.

$$|u - \mathsf{I}_1 u|^2_{H^1(]0,1[)} \leq C(\lambda) \begin{cases} h^2 & \text{for } \lambda > \frac{3}{2}, \\ h^2 |\log(h)| & \text{for } \lambda = \frac{3}{2}, \\ h^{2\lambda-1} & \text{for } \frac{1}{2} < \lambda < \frac{3}{2}. \end{cases}$$

3. (Grading function)

Consider the following Boundary value problem on domain $\Omega =]0,1[$

$$
\begin{aligned}
-\varepsilon^2 u'' + u &= 0 \quad \text{on} \quad]0,1[, \\
u(0) &= 1, \\
u(1) &= \exp(-\frac{1}{\varepsilon}),
\end{aligned}
$$

then
(i) Solve the Boundary Value Problem Analytically.

(ii) Using a heuristic find the grading function for the mesh on which we want to approximate solution u with p.w. linear functions.

(iii) Analize the N-asymptotics of $|u - \mathsf{I}_1 u|_{H^1(\Omega)}$ on the graded mesh given by graded function calculated from part (ii).

Hint for (ii): Grading function g satisfy the following differential equation:

$$g'|u''(g)|^{2/3} = const. \text{ and } g(0) = 0, \quad g(1) = 1$$

4. (Algebraically graded mesh in 2D)

The Algebraically graded mesh is described in the section 5.2.2 for grading factor $(\beta \geq 1)$. Show that

$$\sharp \mathcal{M}_n^\beta \approx n^2$$

(constant independent of n).

(Hint: First prove Lemma 5.2.4)

5. (Geometrically graded mesh)

Geometrically graded mesh for the domain $\Omega =]0,1[$ with n points is described as follows:

for $j \geq 2$ we define $h_j = Cx_{j-1}$ where $C > 0$ is a constant. Than the mesh x_j is define recursively; $x_0 = 0$, $x_j = x_{j-1} + h_j$ and $x_n = 1$

Compute the mesh $\{x_j\}$ in the form of C.

Tutorial: Thursday 10–11 HG E5,

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Testat requirement: 50% of the theoretical exercises and 50% practical exercises (MATLAB) should be solved.